

P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 10 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** the **EIGHT** questions in section **A** and any **FIVE** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae and squared paper are provided.
- Silent, non-programmable calculators should be used.
- State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.
- Clearly indicate the questions you have attempted in a grid on your answer scripts.

Question		Mark
Section A		
Section B		
Total		

SECTION A

1. If $x = \log_a bc$, $y = \log_b ac$ and $z = \log_c ab$. Prove that;
 $x + y + z = xyz - 2$.
2. Given that $y = \tan xy$, show that
$$\frac{dy}{dx} = \frac{y}{\cos^2 xy - x}$$
.
3. Prove that the $\int_0^{\ln 2} \frac{e^x}{1+e^{2x}} dx = \tan^{-1} \frac{1}{3}$
4. The distance of the centre of the circle of radius 5 from the line $3x = 4y$ is 3 units. Find the equation of the tangent to the circle which is parallel to the line $3x = 4y$.
5. Show that the line $\frac{x-2}{2} = \frac{2-y}{1} = \frac{z-3}{3}$ is parallel to the plane $4x - y - 3z = 4$, and find the perpendicular distance of the line from the plane.
6. Find x if $\tan^{-1} x + \tan^{-1} \frac{1}{3} = \frac{\pi}{4}$.
7. The expression $ax^4 + bx^3 - x^2 + 2x + 3$ has a remainder $3x + 5$ when it is divided by $x^2 - x - 2$, find values of a and b.
8. If $y = \sqrt{(5x^2 + 3)}$, show that $\frac{y d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 5$.

SECTION B

Attempt any 5 questions ONLY

9. a) Solve the inequality;
$$\frac{x}{x+1} \leq \frac{x+2}{x+4}$$

b) Given that $f(x) = \frac{\sin^{-1} x}{\sqrt{(1-x^2)}}$, show that $(1-x^2)f''(x) - 3xf'(x) = f(x)$. Hence find the first two non-vanishing terms of the maclaurin's expansion.
10. a) Find $b \int_{3\tan^{-1}4}^{4\tan^{-1}3} \frac{\cos \frac{x}{2}}{4-5\sin \frac{x}{2}} dx$, give your answer to 2 decimal places.
11. (i) Show that $\ln 2^r$ for $r = 1, 2, 3, \dots$ is an arithmetic progression.
(ii) Find the sum of the first 10 terms of the progression.
(iii) Determine the least value of m for which the first 2m terms exceeds 883.7.
12. a) A tangent from the point $T(t^2, 2t)$ touches the curve $y^2 = 4x$. Find

- i) The equation of the tangent
- ii) The equation of the L parallel to the Normal at $(t^2, 2t)$ and passing through $(1,0)$.
- iii) The point of intersection of the line L and the tangent.

b) A point $P(x, y)$ is equidistant from x and T. show that the locus of

$$t^2 - 3t - 2(x + y) = 0$$

13. a) Without using tables, evaluate,

$$\sin \left[\cos^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{-1}{2} \right]$$

b) $\sin 3x + \frac{1}{2} = 2 \cos^2 x$ for $0 \leq x \leq 2\pi$

14. a) Find the Cartesian equation of the plane containing the points $A(2,-1,1)$ $B(1,-2,0)$ and $C(-3,6,1)$. Find the angle between this plane and the line;

$$\frac{x}{4} = \frac{y-1}{1} = \frac{z+3}{5}$$

b) The position vector of points **A** and **B** are $3\mathbf{i} - 8\mathbf{j} + \mathbf{k}$ and $4\mathbf{j} - 2\mathbf{k}$ respectively. Find the position vector of the foot of the perpendicular from the origin 0 to the line **AB**.

15. a) If $(1 + 3i)Z_1 = 5(1+i)$. Show that the locus of $|z - z_1|$ is a circle. Find the coordinates of the centre and radius of the circle.

b) Given that x and y are real, find the values of x and y which satisfy the equation.

$$\frac{2y+4i}{2x+y} - \frac{y}{x-i} = 0$$

16. a) Solve the differential equation

$$\sin x \frac{dy}{dx} + 2y \cos x = 1$$

b) An electric Kettle Switches itself off when the temperature of water in it reaches 100°C at 11:00am when Mr. Nsamba came back and found the temperature of water to be 45°C . 20 minutes later he measured it again and found it to be 65°C . According to the law of heating, the rate of heating of a body in air is proportional to the excess temperature over the surrounding at any time t. if the surrounding temperature was 25°C , Mr Nsamba wants to know the time when the kettle switched off itself.

END