P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 1 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** the **EIGHT** questions in section **A** and any **FIVE** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae and squared paper are provided.
- Silent, non-programmable calculators should be used.
- State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.
- Clearly indicate the questions you have attempted in a grid on your answer scripts.

Question		Mark
Section A		
Section B		
Total		

SECTION A (40 MARKS)
Attempt all questions from this section.

1. If $\frac{\mathbf{2+\sqrt{2}}}{\mathbf{2-\sqrt{2}}} + \frac{\mathbf{1-\sqrt{2}}}{\mathbf{1+\sqrt{2}}} = \boldsymbol{a} + \boldsymbol{b}\sqrt{\mathbf{2}}$ Find the values of a and b. (5 marks)

2. The ninth term of an arithmetic progression is twice the third term, and the fifteenth term is 27. Evaluate the sum of the first 25 terms of the series. (5 marks)

3. Differentiate $x^{\cos x}$ with respect to x. (5 marks)

4. Evaluate the definite integral $\int_0^1 x \tan^{-1} x \, dx$(5 marks)

5. Solve the equation $3\cos 2\theta - 7\cos\theta - 2 = 0$for $0° \leq \theta \leq 360°$. (5 marks)

6. Find the equation of the circle which touches the line $3x - 4y = 3$ at the point $(5, 3)$ and passes through the point $(-2,4)$. (5 marks)

7. The roots of the equation $x^2 + px + 7 = 0$ are $\propto$ and β. Given that $\propto^2 + \beta^2 = 22$,
   find the possible values of p. (5 marks)

8. Prove that $\log_a x = \frac{1}{\log_x a}$. Hence solve the equation $\log_{10} x + \log_x 100 = 3$ (5 marks)

SECTION A (60 MARKS)

Answer any five questions from this section.

9. (a) If $z = x + iy$, determine the Cartesian equation of the locus given by

$$\left|\frac{(z-1)}{(z+1-i)}\right| = \frac{2}{5}$$ (6 marks)

(b) Sketch the loci defined by the equations:

(i) $\arg(z+2) = \frac{-2\pi}{3}$

(ii) $\arg\left(\frac{z-3}{z-1}\right) = \frac{\pi}{4}$

(6 marks)

10. (a) Prove that $\sin 4\theta = \frac{4tan\theta(1-tan^2\theta)}{(1+tan^2\theta)}$(6 marks)

(b) Solve the equation $\tan^{-1}(1+x) + \tan^{-1} 1 - x = \frac{\pi}{4}$ (6 marks)

11. Find the coordinates of any maxima, minima and points of inflexion of the function $y = \frac{3x-1}{(4x-1)(x+5)}$ that it may have. Hence sketch the curve $y = \frac{3x-1}{(4x-1)(x+5)}$

(12 marks)

12. (a) Find $\int x\sqrt{(1-x^2)}\, dx$

(b) Express $\int_0^1 \frac{x^2+x+1}{(x+1)(x^2+1)} dx = \frac{3}{4}\ln 2 + \frac{\pi}{8}$(9 marks)

13. (a) Find the particular solution of the differential equation $xy\frac{dy}{dx} = x^2 + y^2$,

Given that $y = 2,$ when $x = 1$(6 marks)

(b) A lump of radioactive substance is disintegrating. At time t days after it was first observed to have the mass of 10 grams and $\frac{dm}{dt} = -km$ where k is a constant.Find the time, in days for the substance to reduce to 1 gram in mass, given that its half –life is 10 days. (The half – life is the time in which half of any mass of the substance will decay.) (6 marks)

14. (a) Find the values of m for which the line $y = mx$ is a tangent to the circle
$x^2 + y^2 + fy + c = 0$(3 marks)

(b) Find the points where the line $2y - x + 5 = 0$ meets the circle
$x^2 + y^2 - 4x + 3y - 5 = 0$Obtain the equation of the tangents and normal to the circle at these points(6 marks)

15. (a) Show that the points A,B and C with position vectors $2\hat{i} + 3\hat{j}, 4\hat{i} + 5\hat{j}, 6\hat{i} + 9\hat{j}$ respectively are the vertices of a triangle. Find the area of the triangle.
(5 marks)

(b) Find a vector $r$perpendicular to the vectors $s = 5\hat{i} + 3\hat{j} + k$ and
$t = -\hat{i} + 3\hat{j} + 2k$.
Hence, find the equation of a plane passing through the point
$A(5, -1, -2)$ and
parallel to s and t. Find the angle between the plane and the line
$\frac{x-2}{1} = \frac{y-2}{2} = \frac{z-2}{3}$(7 marks)

16. (a) If $y = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ show that $\frac{dy}{dx} = \frac{1}{1+x^2}$ (6 marks)

(b) Use the Maclaurin's theorem to find the first four terms of the expansion of
$e^x \sin x$. (6 marks)