OURLADY OF GOOD COUNSEL SSS

S.6 HOLIDAY WORK 2020

MATHEMATICS P425/1

1. The sum of the first n terms of an A.P is $n^2 + 5n$. Find the first three terms of the series
2. Solve for x. $\log_5(2^x - 1) - \log_5 2 = \log_5\left(\frac{17}{2} + 2^{x-1}\right) - \log_5(2^x - 3)$
3. If a is the first term of an A.P and d is the common difference and L is the last term, prove that the sum is equal to $\frac{1}{2}(a+L)\left(1 + \frac{L-a}{d}\right)$, hence, find the sum of all multiples of 11 between 550 and 1000.
4. Air is pumped into a spherical balloon at a rate of $256\pi\ cm^3 s^{-1}$. When the radius of the balloon is 15 cm, find the rate at which its surface area is increasing
5. Find and classify the nature of the stationary point on the curve $x = 4 - t^3$ and $y = t^2 - 2t$.
6. Given that $y = \sin\theta$ and $x = 1 + \cos 2\theta$, show that $\frac{d^2y}{dx^2} = 4\left(\frac{dy}{dx}\right)^3$
7. Express: $\int_0^{\pi/4} \sin^2 x \cos 2x\, dx$
8. Evaluate: $\int_0^1 \frac{3-x}{(x+1)(x^2+1)}\, dx$
9. Show that $\frac{d}{dx}\left(\tan^{-1} x^x\right) = \frac{(1 + Inx)x^x}{1 + x^{2x}}$.
10. Prove that: $\tan^{-1}\frac{1}{2} - \cos ec^{-1}\frac{\sqrt{5}}{2} = \cos^{-1}\frac{4}{5}$
11. If $\tan\alpha = p$, $\tan\beta = q$, $\tan\gamma = r$, prove that $\tan(\alpha + \beta + \gamma) = \frac{p + q + r - pqr}{1 - pr - rq - pq}$
12. If ABC is a triangle, prove that $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$
13. Solve the equation $\cos 7\theta + \cos 5\theta + \cos 3\theta + \cos\theta = 0$ for $0^o \le \theta \le 360^o$.
14. (a)Find the acute angle between the line whose vector equation is $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \boldsymbol{\mu}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and the plane $\mathbf{r} \cdot (3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$.

(b)Find the position vector of the point where the line b

$\mathbf{r} = -\mathbf{i} - 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ cuts the plane

$\mathbf{r} . (\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = -5$.

15. If the points A, B and C have position vectors $\begin{pmatrix} 1 \\ 3 \\ -5 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 9 \end{pmatrix}$ *and* $\begin{pmatrix} 8 \\ 7 \\ -1 \end{pmatrix}$ respectively. Show that ABC is a triangle.

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