Name..**Stream**.........**Teacher**...................

DEPARTMENT OF MATHEMATICS

S.6 PURE MATHEMATICS–2020

PAPER 1 WEEK 1

3 HOURS

- *Answer **all** the **eight** questions in section **A** and any **five** from section **B**.*
- *Any additional question(s) answered will **not** be marked.*

SECTION A: (40 MARKS)

1. Solve the equation $\sqrt{(6x+1)} - \sqrt{(2x-4)} = 3$. (*05 marks*)

2. Solve $\cos\theta = \sin(\theta + 30^0)$ in the range $0^0 \leq \theta < 360^0$. (*05 marks*)

3. Show that $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{2}\left(1 - \dfrac{x}{4}\right)^{\frac{-1}{2}}$. Write down the first three terms in the binomial expansion of $\left(1 - \dfrac{x}{4}\right)^{\frac{-1}{2}}$ in ascending powers of x. (*05 marks*)

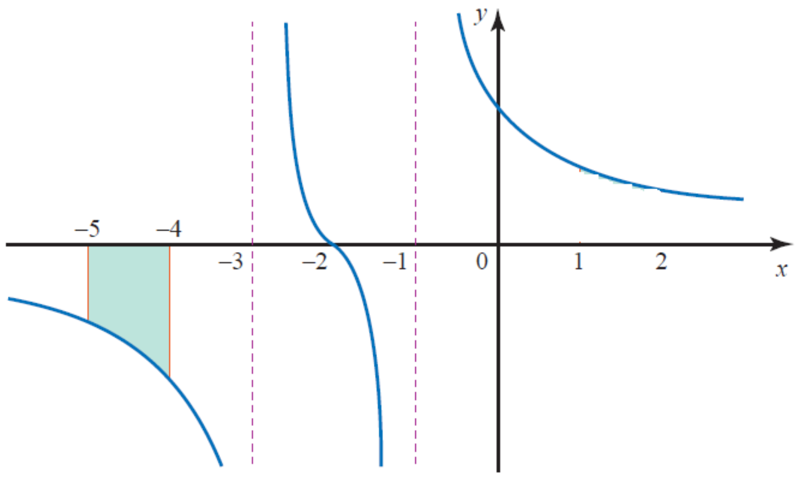
4. Evaluate $\displaystyle\int_0^{\frac{\pi}{8}} \frac{e^{\tan 2x}}{\cos^2 2x} dx$. (*05 marks*)

5. Find the shortest distance from point $P(11, -5, -3)$ to the line l with equation $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix}$. (*05 marks*)

6. An open box with a square base has a total surface area of 300 cm^2. Find the greatest possible volume. (*05 marks*)

7. Find the area of the largest square contained within the circle $x^2 + y^2 - 2x + 4y + 1 = 0$. *(05 marks)*

8. The graph of $y = \dfrac{x+2}{x^2+4x+3}$ is shown below.



Find the area of the shaded region. *(05 marks)*

SECTION B: (60 MARKS)

9. (a) Prove that $\dfrac{d}{dx}(uv) = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$.

(b) Differentiate the following.

(i) $(x-1)^{\frac{1}{3}}(x-2)^3$.

(ii) $\dfrac{2x^2 - 3x}{(x+4)^2}$. *(12 marks)*

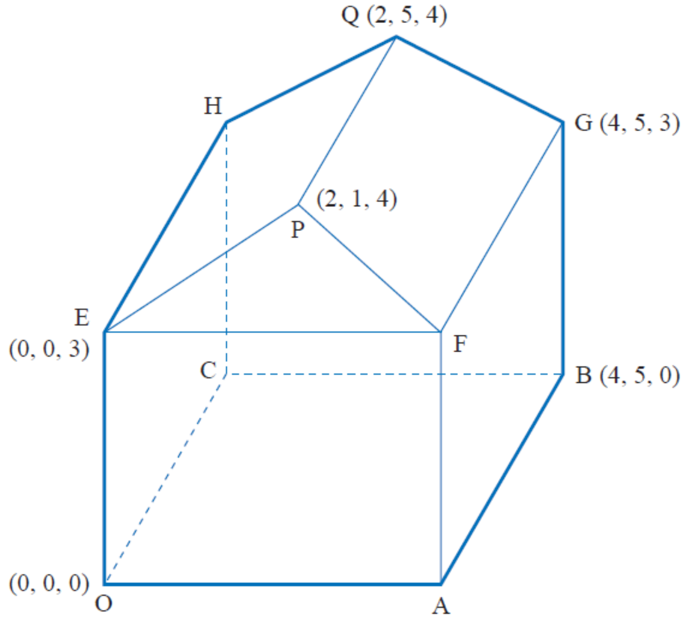
10. (a) Find the equation of the circle passing through the points $A(3, 2)$, $B(-1, 0)$ and $C(5, -2)$. (06 marks)

(b) Show that the locus of the point P with co−ordinates $(1 + 2\cos\theta, 2 + 2\sin\theta)$ is a circle and find its radius and centre. (06 marks)

11. (a) Find $\dfrac{d^2y}{dx^2}$ when $y = \sin^{-1} x - x\sqrt{(1-x^2)}$, expressing your answer as simple as possible.

(b) Use Maclaurin's theorem to express $\ln\sqrt{\dfrac{1+x}{1-x}}$ as a power series up to the term in x^3. *(12 marks)*

12. The diagram shows an extension to a house. Its base and walls are rectangular and the end of its roof, EPF, is sloping, as illustrated.



(a) Write down the co−ordinates of A and F.

(b) Find, using vector methods, the angle EPF.

(c) The owner decorates the room with two streamers which are pulled taut. One goes from O to G, the other from A to H. She says that they touch each other and that they are perpendicular to each other. Is she right? *(12 marks)*

13. Bacteria in a culture increase at a rate proportional to the number of bacteria present. If the number increases from 3000 to 4000 in one hour,

(a) how many bacteria will be present after $2\frac{1}{2}$ hours. *(09 marks)*

(b) how long will it take for the number of bacteria in the culture to become 6000? *(03 marks)*

14. (a) Solve the simulateous equations

$$(x+3)(y+3) = 10 \text{ and } (x+3)(x+y) = 2.$$

(05 marks)

(b) Use the substitution $y = x + \dfrac{2}{x}$ to solve $x^4 - 5x^3 + 10x^2 - 10x + 4 = 0.$ *(07 marks)*

15. (a) Use the substitution $t = \tan\theta$ to solve $\sin 2\theta + 2\cos 2\theta = 1$ for $0 < \theta \leq 2\pi$. *(05 marks)*

(b) The equation $1 + \sin^2\theta^0 = a\cos 2\theta^0$ has a root of 30. Find the value of a and all the roots in the range 0 to 360. *(07 marks)*

16. (a) Prove by induction that $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. *(05 marks)*

(b) Aisha opens an account with a saving scheme which offers a 12.9% compound interest per annum. The scheme does not allow any withdrawal until after a period of 5 years. A customer is required to deposit a half the amount of money he/she opens the account with every beginning of other years for a period of 4 years. If Aisha started with shs 600,000, calculate how much;

(i) money she will earn from the scheme after 5 years.

(ii) interest she earns from the saving scheme. *(07 marks)*

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