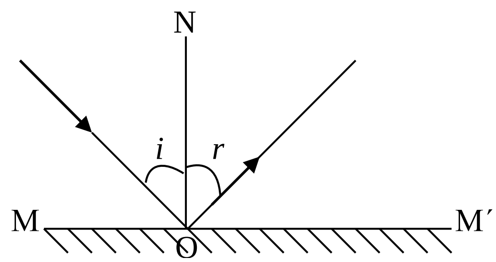
1. *REFLECTION*



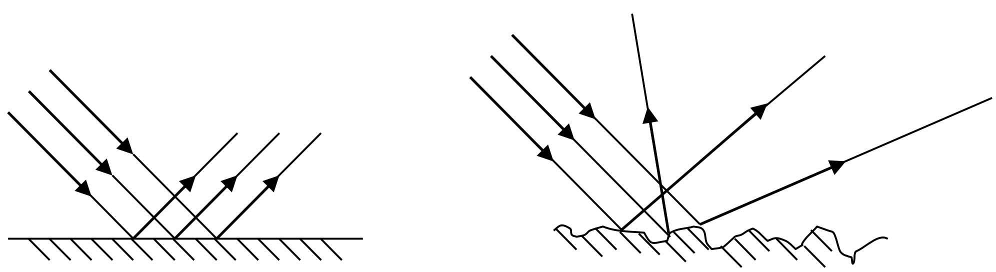
Line NO is the normal to the mirror MM´.
i is the angle of incidence and *r* the angle of reflection

1.1: Laws of light

1. The reflected ray, the incident ray, and the normal to the mirror at the point of incidence all lie in the same plane.
2. The angle of incidence is equal to the angle of reflection

1.2: Regular and Diffuse Reflection.

Diffuse reflection is said to occur when a regular beam of light gets scattered on reflection by an uneven reflecting surface

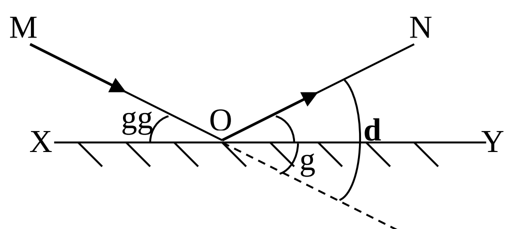


(i) Regular

(ii) Diffuse

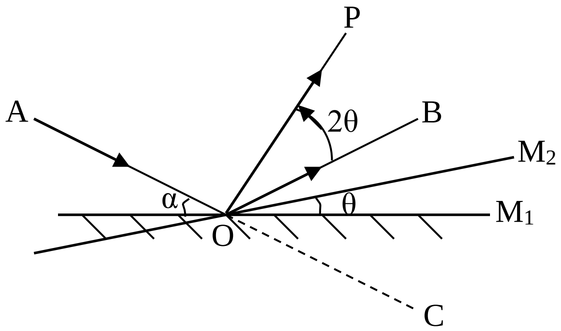
In both, the laws of reflection are obeyed. For diffuse reflection the rays in the parallel incident beam of light strike the surface at various angles of incidence. Thus they are accordingly reflected in various directions.

1.3: Deviation of Light by a Plane Mirror



g = glancing angle.
Consider a ray MO incident on the mirror at a glancing angle g. It is reflected along ON. Since the angle of incidence is equal to the angle of reflection ∠NOY = g. So that the deviation, d = 2g

1.4: Deviation of Reflected Ray by Rotated Mirror



Consider a ray AO incident at O on a plane mirror M_1, at a glancing angle α.
If OB is the reflected ray, then angle BOC = 2α.
Suppose M_1 is now rotated through an angle θ to position M_2, the direction of AO remaining constant. Then the glancing angle becomes $\alpha + \theta$ and $\angle POC = 2(\alpha + \theta)$. Thus the reflected ray has rotated through an angle $POB = \angle POC - \angle BOC$

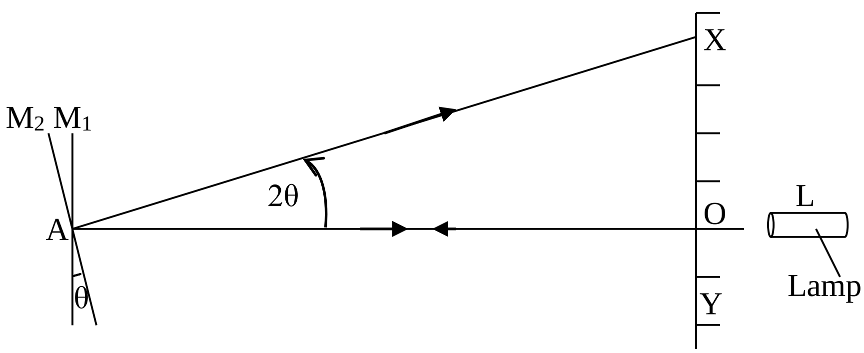
$$= 2(\alpha + \theta) - 2\alpha = 2\theta.$$

Thus the reflected ray moves through twice the angle turned through by the mirror.

1.5: Applications of Reflection at Plane Surfaces

1. Optical Lever in Mirror Galvanometer

Light is used as a weightless pointer. In such instruments a small mirror M_1 is rigidly attached to a system which rotates when a current flows in it.

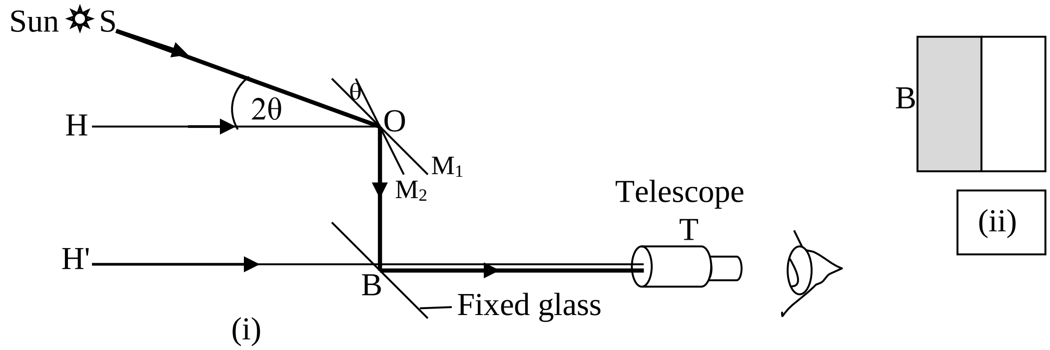


A beam of light from a fixed lamp L is directed on to the mirror. When the beam is normal to M_1, it is reflected directly back and a spot of light is obtained at O on scale Y (just above L). This is when there is no current flowing in the coil of the galvanometer.

If a current passes through the system so that the system rotates by θ, the reflected beam rotates through 2θ, thus making the system sensitive.

2. The sextant

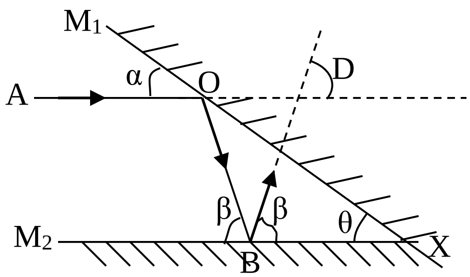
It is an instrument used for measuring the angle of elevation of the sun or stars.



B is plane glass silvered on a vertical half as shown in figure (ii).
Looking through the telescope T, the mirror O is turned about a horizontal axis until the view H' of the horizon seen directly through the unsilvered half of B, and also a view of it, H, seen by successive reflection at O and the silvered half of B, are coincident. Mirror O is then parallel to B in position M_1. The position M_1 is noted. O is now rotated until a position M_2 in which the image of the sun S is seen on horizon H'. The angle of rotation, θ, between positions M_1 and M_2 of O is found.
The elevation SOH of the sun S is equal to 2θ.

1.6: Successive Reflection at Two Plane Mirrors

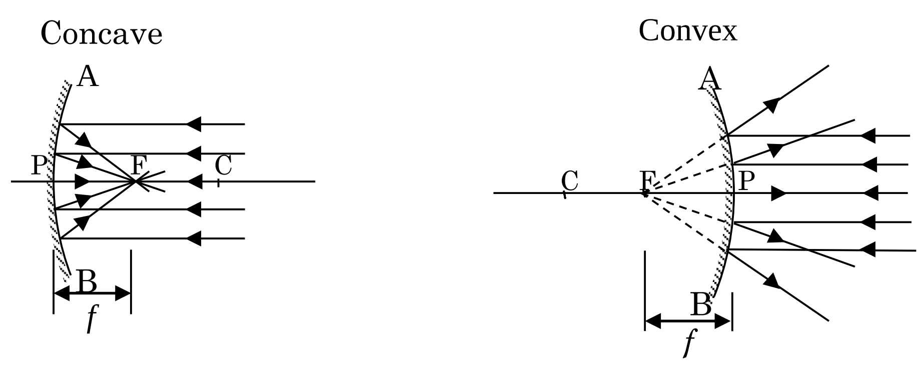
A ray AO, incident at O on mirror M_1 is deviated clockwise. On striking mirror M_2 it is deviated anticlockwise so that the net deviation is D



$$
\begin{aligned}
D &= 180° - [180° - 2\beta] - 2\alpha \\
&= 2\beta - 2\alpha \\
&= 2\theta
\end{aligned}
$$

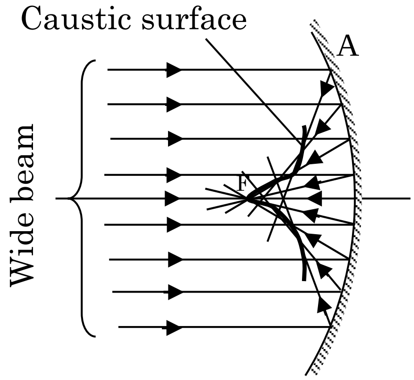
Exercise:
Two plane mirrors A and B are set vertically in a room, facing each other and 8.0 m apart. A candle is set 1.5 m from A. An observer in the middle of the room sees two distinct images of the candle. Determine the distances of these images from the observer. [**Ans: 6.5m, 10.5m**]

2. CURVED REFLECTORS



For a curved mirror, a secondary axis is a straight line through the centre of curvature but not through the pole of the mirror.

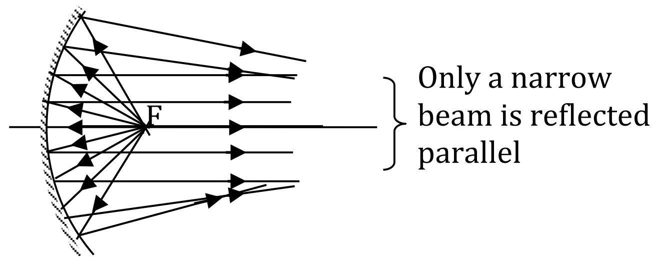
2.1: Caustic Surface



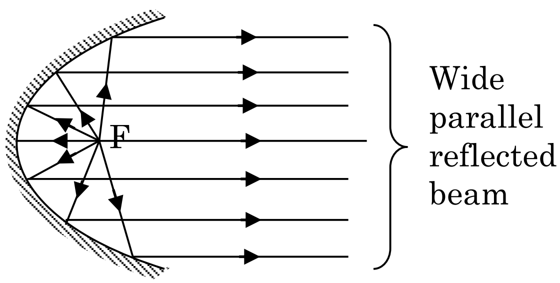
When a wide beam of light, parallel to the principle axis, is incident on a concave mirror the reflected rays do not pass through a single point as a narrow beam does. The subsequent reflected rays meet at other points before the principal axis. The locus of such points forms A bright surface known as the ***caustic surface*.**

2.2: Parabolic versus Spherical Mirror

(i) Spherical



(ii) Parabolic reflector

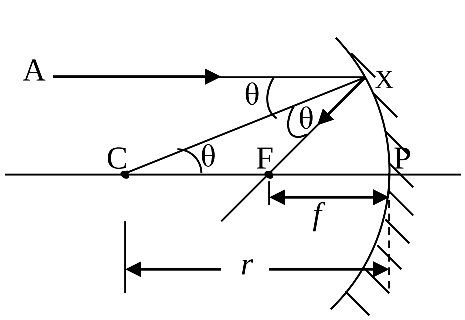


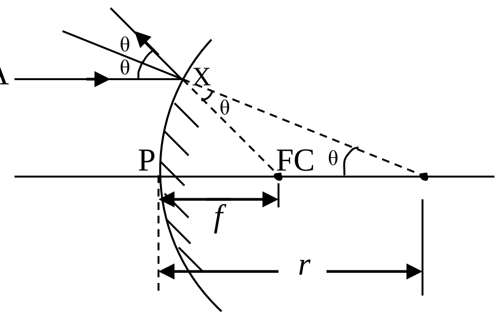
A parabolic mirror is capable of producing a wide parallel beam of light when the source of light is placed at its principal focus. If the source were to be placed at the principal focus of a spherical mirror, only those rays which strike near the pole are reflected as a parallel beam. The rest are reflected divergently. For this reason, parabolic mirrors are preferred where a strong reflected beam is required e.g in search lights, vehicle head lamps, etc.
Further, if a wide parallel beam of light is incident on a parabolic reflector, it is brought to a single focus. This is why they are employed in reflector telescopes

2.3: Focal Length and Radius of Curvature

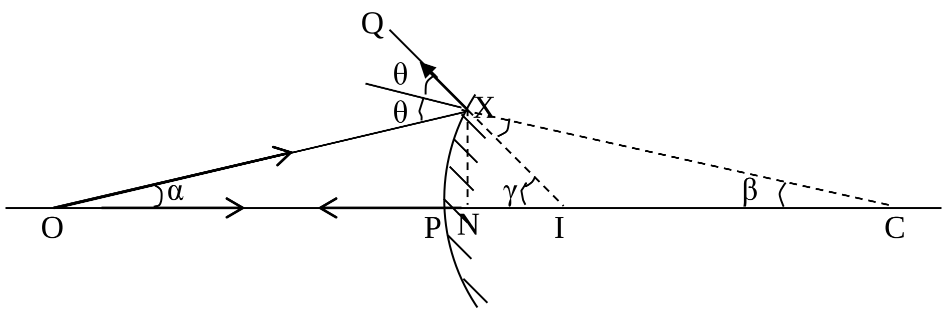
Consider a ray AX parallel and close to the principal axis of the mirror. It is reflected through F in if we take the case of a concave reflector, If C is the centre of curvature, then CX is the

normal to the mirror at X. Thus CF = FX. Since AX is close to the principal axis, X is very close to P and FX is approximately equal to FP so that FP = FC or FP =½CP
Therefore **f = r/2**





2.4: The Mirror Formula



Consider a point object O on the principle axis of a convex mirror. A ray OX from O is reflected along XQ. A ray OP, incident at P, is reflected back along PO and the point I where the two rays appear to emerge from is the virtual image of O. From the geometry of the figure

$$\theta = \alpha + \beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Also $\theta = \gamma - \beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$

Therefore $\gamma - \beta = \alpha + \beta$

$$\gamma - \alpha = 2\beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Now, $\gamma = h/IN = h/(-IP)$ as I is virtual.

$\alpha = h/ON = h/(+OP)$ as O is real

$\beta = h/NC = h/(-PC)$ as C is virtual

Substituting for α, β and γ in (3)

$-h/(-IP)- h/(+OP) = 2h/(-CP)$

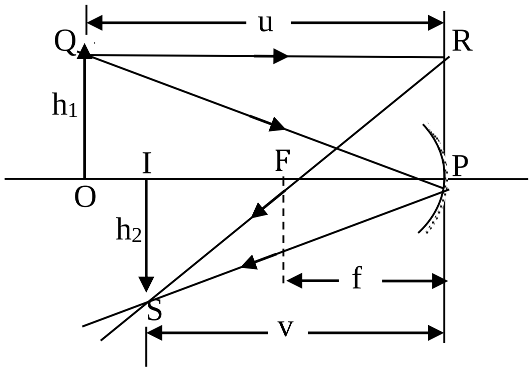
So $1/IP + 1/OP= 2/CP$

$1/v + 1/u = 2/r$

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$

Qn. Derive the same formula using a concave mirror.

ALTERNATIVE DERIVATIONS
Concave Mirror



Imagine an object OQ of height h_1 at O.
A ray QR parallel to the principal axis is reflected through F, the principal focus. A ray QP, incident at the pole, is reflected through S such that ∠QPO = ∠SPI and the point, S, where the two reflected rays meet is the image of Q. Also I is the image of O since O is on the principal axis, and IS is the image of OQ.
Now ΔQPO is similar to ΔSPI

So $\frac{h_2}{h_1} = \frac{IP}{OP} = \frac{v}{u} \ldots\ldots (1)$

And ΔSIF is similar to ΔRPF

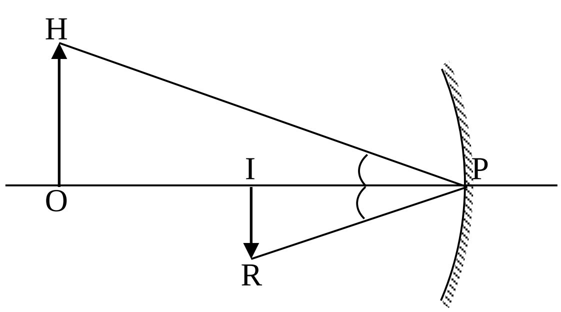
So $\frac{SI}{PR} = \frac{IF}{PF}$ which leads to $\frac{h_2}{h_1} = \frac{v-f}{f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$

From (1)and (2) $\frac{v}{u} = \frac{v-f}{f}$

Deviding through by v and rearranging, we have $\mathbf{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$

2.5: Lateral Magnification (m)

$$m = \frac{\text{Height of image}}{\text{Height of object}} = \frac{IR}{OH}$$



From the similarity of triangles OPH and IPR,

$$\frac{IR}{OH} = \frac{IP}{OP} = \frac{v}{u}$$

$\therefore \quad m = \frac{v}{u}$ Now $\quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\therefore 1 + \frac{v}{u} = \frac{v}{f}$$

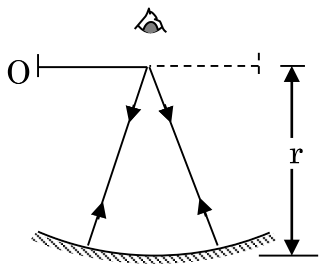
$$\therefore \quad \mathbf{m = \frac{\mathit{v}}{\mathit{f}} - 1}$$

2.6: Determination of Focal Length and radius of curvature

Concave Mirror

No parallax method

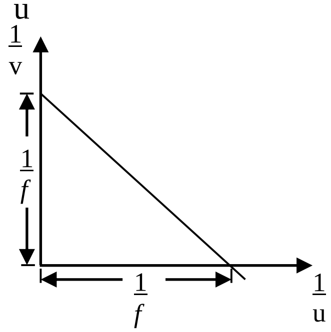
‘No parallax’ means no relative motion, in this case between the object and its image. When this happens, the two are in the same place.



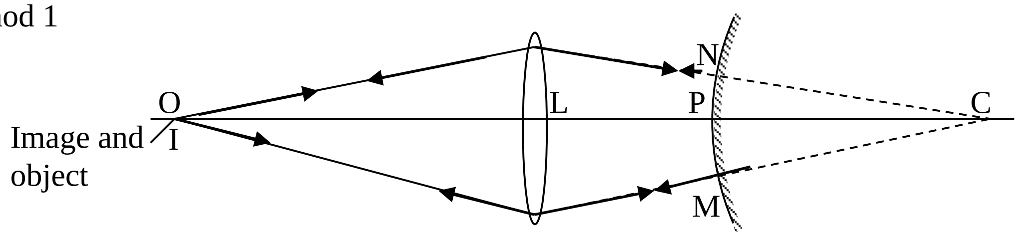
The mirror under test is placed on a bench facing up
A horizontal pin is held above the mirror and while observing from above the height of the pin is adjusted until it is observed to be in the same position with its image. This is confirmed if when the observer moves his eye across the pin, the two move together without separating. The distance between the pin and the mirror is the radius of curvature, r. So the focal length, f = r/2, is found

Method 2: Using the mirror formula

Using the method of no parallax, or employing an illuminated object, several values of the image distance, v, are obtained, corresponding to several values of the object distance u.
A graph of $\frac{1}{v}$ against $\frac{1}{u}$ is plotted. The intercept gives $\frac{1}{f}$

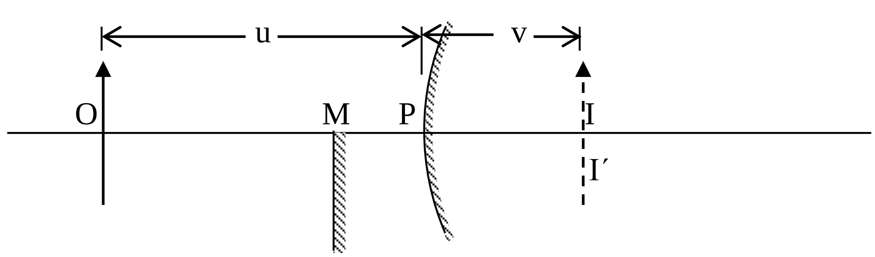


Convex Mirror
Method 1



Using a convex lens L, a real image of an illuminated object O is formed at point C. Distance LC is noted. The convex mirror is then place between L and C with its reflecting surface facing the lens and is moved along the axis OC until a real image of O is formed at O. Distance LP is noted. Under these conditions the rays from O must be striking the mirror normally eg at M and N.
Thus PC = r, the radius of curvature
Now PC = LC – LP
∴ r = LC – LP

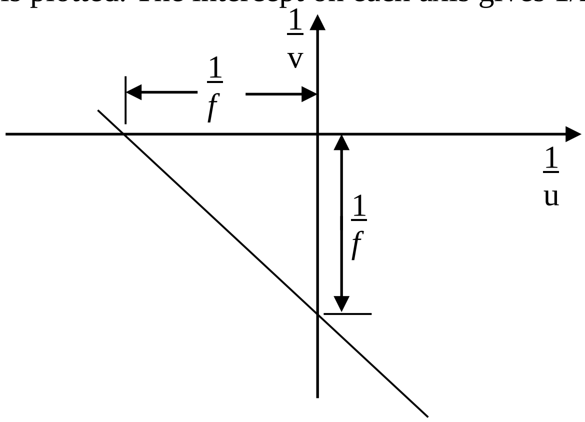
Method 2



A pin O is placed to form an image I in the convex mirror. Then a small plane mirror, M, facing O is moved between O and P until the image, I′, of the lower part of O coincides with I. The distances OP and MP are measured. Due to the plane mirror, OM = MI

$\therefore v = OM - MP$ (virtual) and $u = OP$ (real)

The procedure is repeated for several positions of O each time working out u and v. A graph of 1/v against 1/u is plotted. The intercept on each axis gives 1/f



Worked Examples

1. A concave mirror forms, on the screen, a real image three times the linear dimensions of the object. The object and the screen are then moved until the image is twice the size of the object. If the shift of the screen is 25 cm, determine

(I) the focal length of the mirror

(ii) the shift of the object

Solution:

(i) Let u = original object distance

f = focal length of the mirror

x = shift of the object

Then, using $m = \frac{v}{f} - 1$ for the two cases, we have

$$3 = \frac{3u}{f} - 1 \qquad \therefore 4f = 3u \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

and $$2 = \frac{(3u - 25)}{f} - 1 \qquad \therefore 3f = 3u - 25 \quad \ldots\ldots\ldots\ldots..(2)$$

Eqn (1) – eqn (2): $f = \underline{25\text{ cm}}$

(ii) From eqn(1) $u = 4f/3 = 100/3$ cm

Since the image becomes smaller, the screen was moved closer to the mirror as the object was shifted away from the mirror. So for the second case

$$\frac{3u - 25}{u + x} = 2$$

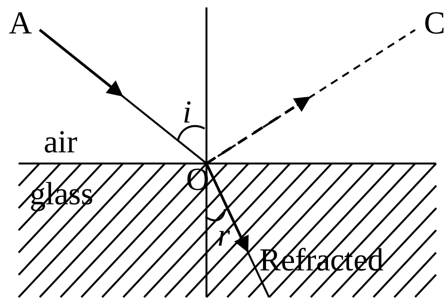
$\therefore 3u - 25 = 2u + x$

$\therefore x = u - 25 = (100/3) - 25 = \underline{4.16\text{ cm}}$

3. REFRACTIION AT PLANE SURFACE

Refraction is the change of direction of travel of light resulting from change of speed when light crosses from one medium to another of different optical density.

3.1: Laws of Refraction



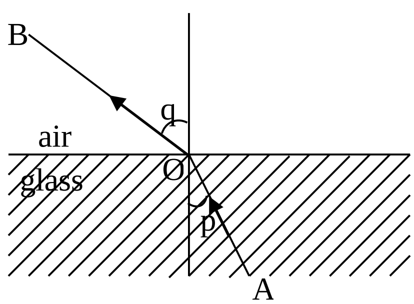
1. The incident ray, the refracted ray and the normal at the point of incidence, all lie in same plane.
2. Snell's law: For two given media, $\frac{\sin i}{\sin r}$ is a constant, where i is the angle of incidence and r the angle of refraction.

The ratio $\mathbf{\frac{\sin i}{\sin r}}$ is known as the ***refractive index*** for the two given media

If the incident ray is in vacuum the ratio $\frac{\sin i}{\sin r}$ is called the ***absolute refractive index***

For air the refractive index is approximately 1.00028. So for practical purposes air is regarded as vacuum.

$$\text{Refractive index,}\quad n = \frac{\text{velocity of light in a vacuum}, c}{\text{velocity of light in medium}, v}$$



$_gn_a = \frac{\sin p}{\sin q}$ by definition

From the principle of reversibility of light a ray travelling along BO in air is refracted along OA in the glass.

Then $_an_g = \frac{\sin q}{\sin p}$

Thus

$$\boxed{\mathbf{_gn_a = \frac{1}{_gn_a}}}$$

Consider a ray PQ incident in air on a plane glass boundary and finally emerging along a direction RS in air. If the boundaries of the media are parallel, RS is parallel to PQ.
Let i_g, i_w respectively be the angles made with the normals in glass and water media

Then, $_gn_w = \frac{\sin i_g}{\sin i_w}$

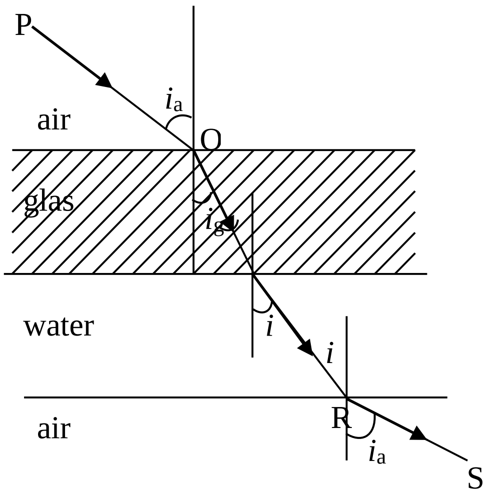
But $\frac{\sin i_g}{\sin i_w} = \frac{\sin i_g}{\sin i_a} \times \frac{\sin i_a}{\sin i_w}$

and $\frac{\sin i_g}{\sin i_a} = {}_g n_a$ and $\frac{\sin i_a}{\sin i_w} = {}_a n_w$

therefore ${}_g n_w = {}_g n_a \times {}_a n_w$

Now ${}_g n_a \times {}_a n_w = \frac{v_g}{v_a} \times \frac{v_a}{v_w} = \frac{v_g}{v_w} = {}_g n_w$

$$\therefore \boxed{{}_g n_w = \frac{{}_a n_w}{{}_a n_g}}$$



General Relationship between n and sin i

$\sin i_a = {}_a n_g \sin i_g$

Also $\sin i_a = {}_a n_w \sin i_w$

Hence $\sin i_a = {}_a n_g \sin i_g = {}_a n_w \sin i_w$

Since $n_a = 1$, ${}_a n_g = n_g$ and ${}_a n_w = n_w$, we can write

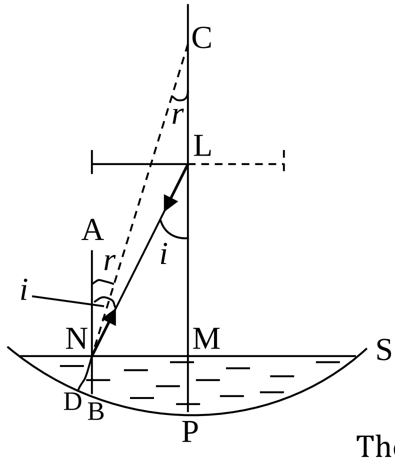
$n_a \sin i_a = n_g \sin i_g = n_w \sin i_w$

Therefore **$n\sin i$ = constant**

3.2: Refractive Index of a liquid by using a concave mirror

- A concave mirror,S, is placed on a bench.
- A pin is held above the mirror and a position along the principal axis is found where it coincides with its own image. The height of the pin from the pole of the mirror is measure and noted. It is equal to the radius of curvature of the mirror.
- A little of the liquid is placed on a concave mirror and a position L is located by the no-parallax method where the image of a pin held over the mirror coincides in position with the pin itself. The distance LP, between the pin and the mirror is measured.

In this case the rays are reflected back along the incident path and must therefore be striking the mirror normally.



A ray LN close to the axis LP is refracted at N along ND in the liquid, strikes the mirror normally at D, and is reflected back along DNL.
Thus if DN is produced it passes through the centre of curvature C.
Let ANB be the normal to the liquid surface at N.
Then ∠ANL = ∠ NLM = *i* (angle of incidence)
and ∠BND = ∠ANC = ∠NCM = r (angle of refraction)

$$\text{The refractive index, n} = \frac{\sin i}{\sin r} = \frac{NM/LN}{NM/CN} = \frac{CN}{LN}$$

Since LN is a ray very close to the principle axis CP, LN is approximately = LM and CN = CM so that

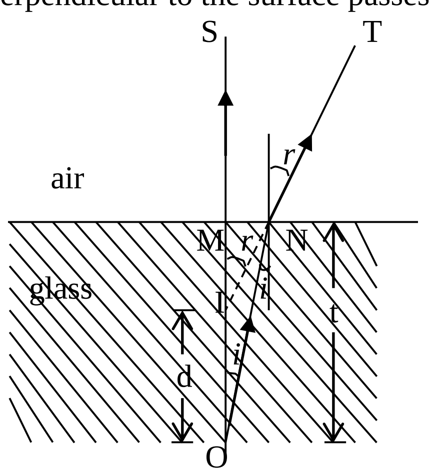
$$n = \frac{CM}{LM}$$

But if the depth MP of the liquid is very small compared with LM and CM, CM = CP and LM = LP approximately.

Hence, approximately, $n = \frac{CP}{LP}$, where CP is the radius of curvature.

3.2: Apparent Depth

Consider an object O below the surface of the medium of refractive index n. A ray OM from O perpendicular to the surface passes straight into the air along MS.



A ray ON, very close to OM, is refracted at N away from the normal along NT so that to an observer directly overhead the object O appears to be at I.

Now, $n\sin i = 1 \times \sin r$

i.e $n = \frac{\sin r}{\sin i} = \frac{MN/IN}{MN/ON} = \frac{ON}{IN}$

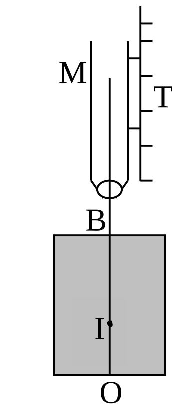
Since the observer is directly above O, the rays ON and IN are very close to the normal OM.
Hence ON is approximately equal to OM and IN = IM.

Thus $n = \frac{OM}{IM}$

$$\textbf{Therefore} \qquad \mathbf{n} = \frac{\textbf{real depth}}{\textbf{apparent depth}}$$

If OM = t, the apparent depth = t/n, the displacement, d = t – t/n = **t(1- 1/n)**

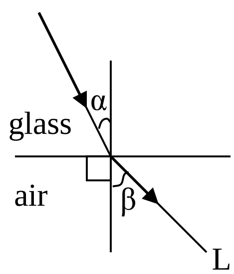
Refractive Index by Apparent Depth Method

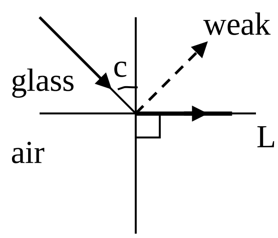


A traveling microscope M is focused on lycopdium particle on a sheet of white paper and the reading on the scale t is noted.
Suppose it is x cm. If the refractive index of glass is required, a parallel-sided glass slab is placed on the paper, and M is adjusted until the particles are focused at I. Let the reading at T be y cm.
Some lycopodium particles are then sprinkled on top of the glass slab and S is raised until they are focused. Let the reading on T be z cm
Then real depth of O = OB = (z-x) cm
Apparent depth = IB = (z-y) cm

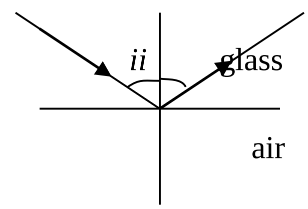
$$n = \frac{\text{real depth}}{\text{apparent depth}} = \frac{z-x}{z-y}$$

3.3: Total internal reflection





c = critical angle

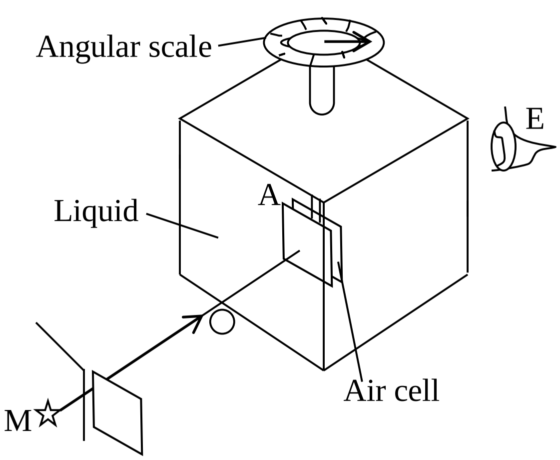


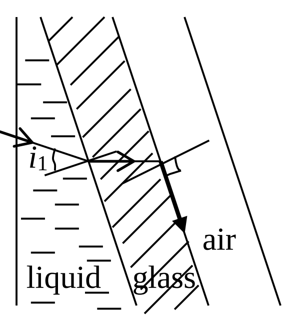
Total internal reflection

$n\sin c = 1 \times \sin 90^o$
$\sin c = 1/n$

Examples of Total internal Reflection: Mirages, formation of rain bow (here dispersion also occurs)

3.4: Refractive Index of a liquid by Air-cell Method





An air cell is formed by cementing together two thin plane-parallel glass plates so as to contain a thin film of air of constant thickness.
The liquid is placed in a glass vessel having thin plane-parallel sides. The air cell A is placed in the liquid. Bright light from a source, M is directed to one side of A in a constant direction MO, and is observed at E on the other side. A is first positioned so that the incident light from M strikes it normally and goes through undeviated. A is now rotated (slowly) until the light is suddenly cut off from E.

The angle, i_1, turned through is noted. It is the angle of incidence in the liquid when light just grazes the glass-air boundary.
Since the boundaries are parallel nsin i =constant

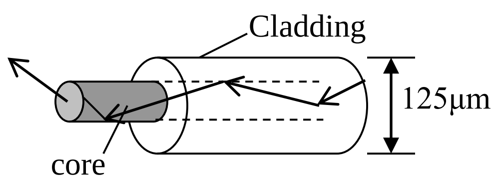
$\therefore n_1 \sin i_1 = n_g \sin i_2 = 1 \times \sin 90°$, where n_1 is the refractive index of the liquid

$\therefore \quad n_l \sin i_1 = 1$

$$n_l = \frac{1}{\sin i_1}$$

3.4: Optical Fibre

This is an application of total internal refraction



It is a very fine glass rod of diameter about 125µm with a central glass core surrounded by a coating (CLADDING) of smaller refractive index.
A beam of light entering one end of the core is totally internally reflected several times until it comes out at the other end. There are many advantages of transmitting information by optical fibre instead of copper wires. E.g

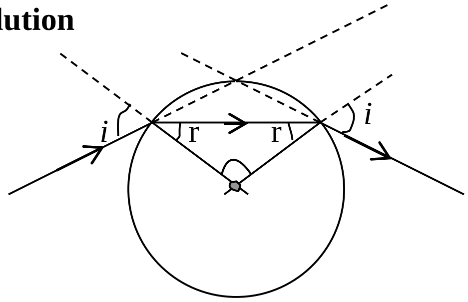
(i) It has a high information – carrying capacity.
(ii) It suffers no electric interference.
(iii) Can cover greater distances i.e fewer repeaters
(iv) Opticalfibre cables are lighter and smaller
(v) Cheaper to produce and easier to maintain since suffers NO corrosion.
(vi) Signals travel faster since light travels faster than electrons

Examples:

1. A ray of light is refracted through a sphere, whose material has refractive index n, in such away that it passes the extremities of two radii which make angle α with each other. Prove that if γ is the deviation of the ray.

$\cos\frac{1}{2}(\alpha-\gamma) = n\cos\frac{1}{2}\alpha$

Solution



2r = 180 – α

r = 90 - ½α

Also γ = 2i - 2r

Therefore i = ½γ + r = 90 - ½(α-γ)

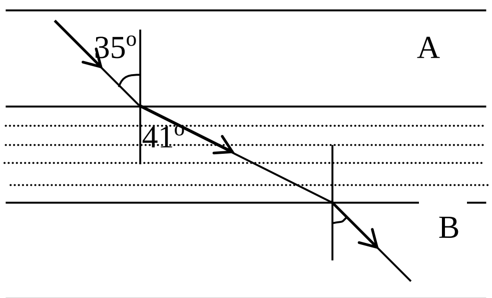
Now sin i = nSin r

∴ sin [90 - ½(α -γ)] = nSin (90 ½α)

∴ cos½(α-γ) = ncos½α

∴ cos½(α-γ) = ncos½α

2. The figure below shows a layer of liquid confined between two transparent parallel plates A and B of refractive indices 1.54 and 1.44 respectively.



A ray of monochromatic light making an angle of 35° with the normal for the interface between medium A and the liquid is refracted through an angle of 41° by the liquid. Find

(i) the refractive index of the liquid.

(ii) the angle of refraction in medium B

(iii) the minimum angle of incidence in medium A for which the light will not emerge from medium B.

Solution:

(i) $n_A \sin i_A = n_l \sin i_l$

$$\therefore n_l = \frac{n_A \sin i_A}{\sin i_l} = \frac{1.54 \sin 35^o}{\sin 41^o} = \underline{1.35}$$

(ii) $n_A \sin i_A = n_B \sin i_B$

$$\therefore \quad \sin i_B = \frac{n_A \sin i_A}{n_B} = \frac{1.54 \sin 35^o}{1.44} = 0.613$$

$$\therefore \quad i_B = \underline{37.8^o}$$

(iii) The angle in medium B = Critical angle of B

Let c_B = critical angle for medium B

Then $\sin c_B = 1/n_B$

Now $n_A \sin i_A = n_B \sin c_B = n_B/n_B = 1$

$\therefore \quad \sin i_A = 1/n_A = 0.649 \qquad \therefore i_A = \underline{40.5^o}$

Exercises

1. A ray of monochromatic light is incident on a parallel-sided glass block of width ***w*** placed in a vacuum. The ray suffers a deviation β. Given that the original direction of the ray makes an angle α with the first face of the block and the refractive index is **n**, show that the light takes time **t** to emerge from the opposite face, where

$$t = \frac{nw\,\mathrm{cosec}(\alpha + \beta)}{c}$$

and c is the velocity of light in vacuum.