

NAME.....STREAM.....GROUP.....



KING'S COLLEGE, BUDO

END OF TERM III EXAMINATIONS

S.5 MATHEMATICS–P425/1

PAPER 1

3 HOURS

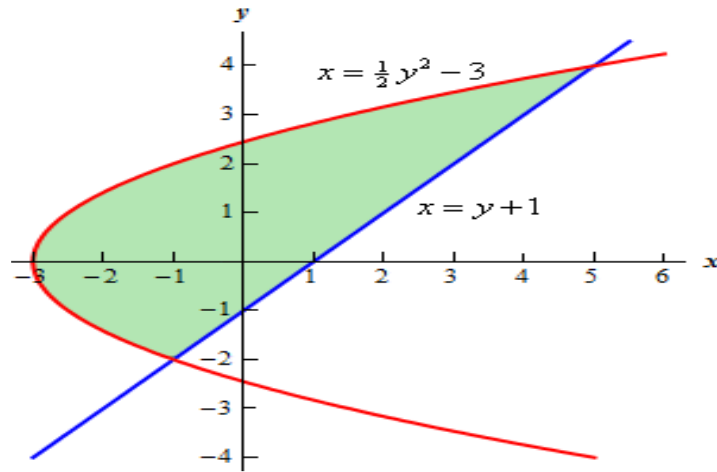
INSTRUCTIONS

- ❖ Answer **all** the questions in section **A** and any **five** from section **B**.
- ❖ Write your **name** and **stream** on every answer sheet used.
- ❖ All working **MUST** be logically shown.
- ❖ Any additional question(s) answered will **NOT** be marked.

	SECTION A								SECTION B					
QNS	1	2	3	4	5	6	7	8						TOTAL
MKS														

SECTION A: (40 MARKS)

1. Solve the equation $3\sqrt{t} - \sqrt{(t+5)} = 3$. (05 marks)
2. If $y = ae^{-2x}\sin(x + \beta)$ where a and β are constants, prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$. (05 marks)
3. Show that $\tan^{-1}\left(\frac{x-y}{1+xy}\right) + \tan^{-1}\left(\frac{y-z}{1+yz}\right) = \tan^{-1}\left(\frac{x-z}{1+xz}\right)$. (05 marks)
4. Find the area enclosed by the curve $x = \frac{1}{2}y^2 - 3$ and the line $x = y + 1$ below:



5. Find $\int e^x \cos x \, dx$. (05 marks)
6. How many terms of the series $1+8+15+\dots$ are required to give a sum of 5500? (05 marks)
7. If A is the point (2, 0) and B is the point (-3, 0), find the locus of a point P which moves so that $AP^2 + 2BP^2 = 22$. (05 marks)
8. Find the values of x and y in $\frac{x}{2+3i} - \frac{y}{3-2i} = \frac{6+2i}{1+8i}$. (05 marks)

SECTION B: (60 MARKS)

9. (a) If $\sin(x + \alpha) = 2\cos(x - \alpha)$, prove that $\tan x = \frac{2 - \tan \alpha}{1 - 2 \tan \alpha}$. Hence solve $\sin(x + 45^\circ) = 2\cos(x - 45^\circ)$ for $-2\pi \leq x \leq 2\pi$.
 (b) Find the maximum and minimum values of $3\sqrt{2} \cos(\theta + 45^\circ) + 7 \sin \theta$ by expressing in the form $R \cos(\theta - \alpha)$. State the smallest values of θ for which they occur. (12 marks)
10. A curve has the equation $y = \frac{x^2 - 4x}{x^2 - 4x + 3}$
 - (i) Determine the nature of the turning point on the curve.
 - (ii) Determine the equations of the asymptotes of the curve.
 - (iii) Sketch the curve. (12 marks)

11. (a) Use the substitution $u = e^{2t}$ to find $\int \frac{e^{2t}}{1+e^{4t}} dt$.
- (b) Evaluate $\int_0^1 \frac{3y-1}{y^2+3y+2} dy$ correct to 4 decimal places. (12 marks)
12. (a) Find the equation of the tangent to the curve $\frac{x^3-y}{1-y^3} = x$ at the point $(1, -1)$.
- (b) Differentiate the following with respect to x
- (i) $(\tan^{-1} x)^{x^2}$ (ii) $\left(\frac{x-2}{x^2-3}\right)^3$. (12 marks)
13. (a) The roots of the equation $ax^2 + bx + c = 0$ are α and β .
- (i) Find the values of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ and $(\alpha - \beta)^2$.
- (ii) Express $\frac{c}{b}$ in terms of α and β .
- (b) The polynomial $x^4 + ax^3 + bx^2 + cx - 12$ is divisible by both $(x - 2)$ and $(x + 2)$. Find the values of a and b . (12 marks)
14. (a) Prove that $\log_q p = \frac{\log_r p}{\log_r q}$. Hence given that $\frac{\log_{10} 3}{\log_{10} 2} = \frac{8}{5}$ evaluate $(\log_3 2)^2$.
- (b) Find x in the equation $\log_2 x - \log_x 8 + 2 = 0$.
- (c) Simplify $\frac{\log 81}{\log 3} - \frac{\log 9}{\log 27}$. (12 marks)
15. (a) Given that $y = \ln(x^2 + x + 2)$, find the Maclaurin's expansion of y up to the third term. Hence estimate $\ln 2.11$ truncated to 4 significant figures.
- (b) The coefficient of x^2 in the expansion of $(1 - 2x)^n$ is 24. Calculate the value(s) of n . (12 marks)
16. (a) Find the Cartesian equation of the locus of a point P represented by the equation
- $$\left| \frac{z+2-i}{z-1} \right| = 2.$$
- (b) Solve the equation $z^3 + 8 = 0$. (12 marks)

–END–

SUCCESS