Name..**Stream**.........**Teacher**..................

DEPARTMENT OF MATHEMATICS

S.6 PURE MATHEMATICS−2020

PAPER 1 COVD-19 WEEK 2

3 HOURS

- *Answer **all** the **eight** questions in section **A** and any **five** from section **B**.*
- *Any additional question(s) answered will **not** be marked.*
- *Each number has been motivated with some hints.*

SECTION A: (40 MARKS)

1. Solve the equation $16\log_x 2 + \log_2 x = 10$. (*05 marks*)
Hint: Use change of base.

2. Prove that $\tan(x + 45^0) + \tan(x - 45^0) = 2\tan 2x$. (*05 marks*)
Hint: Preferably go via the L.H.S. Use compound and double angle formulaes for tan.

3. Given that $x^4 + bx + c$ is divisible by $(x - 2)^2$, find the values of b and c. (*05 marks*)
Hint: Recall that for repeated factors $f(x) = 0$ and $f'(x) = 0$.

4. Show that $\int_0^{\frac{\pi}{4}} x\sin 5x \sin 3x\ dx = \frac{\pi - 2}{16}$. (*05 marks*)
Hint: Apply $c - c = -2ss$(factor formula for cos) then switch to integration by parts.

5. Find l, the line of intersection of the two planes $3x + 2y - 3z = -18$ and $x - 2y + z = 12$. (*05 marks*)
Hint: There are several methods you can employ. Use any convinient.

6. An error of 0.25% was made in measuring the radius of a sphere. Calculate the corresponding percentage increase in its surface area. *(05 marks)*
Hint: An application of small incerements. See Backhouse 1.

7. Find the equation of a parabola having focus at $(0, -4)$ and vertex $(0, 2)$. *(05 marks)*
Hint: Recall that the distance from vertex to focus is equal to the distance from vertex to the directrix and apply distance formulaes.

8. Find the particular solution of the differential equation $(x^2 - y^2)\frac{dy}{dx} = xy$ given that $y = 2$ when $x = 4$. *(05 marks)*
Hint: Use the substitution $y = ux$(Homogeneous o.d.e).

SECTION B: (60 MARKS)

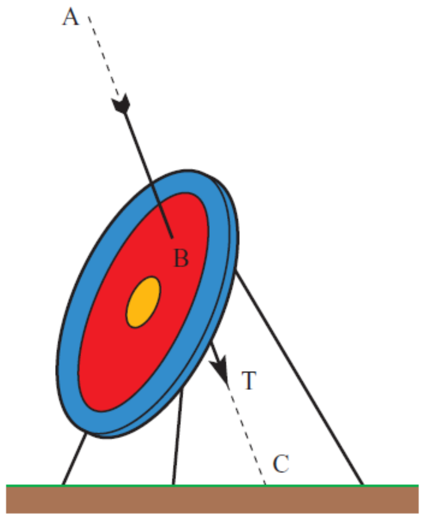
9. Investigate the stationary values of $\frac{x^3}{1+x^2}$ and sketch the graph of $y = \frac{x^3}{1+x^2}$. (12 marks)
Hint: Use the $TIAT$ procedure for curve sketching. Rememeber T =Turning points and their nature, I =Intercepts, A =Assymptotes and T =Table of critical values. Also recall the features of curves with slant/oblique assymptotes.

10. Show that the line $y = mx + c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$. Hence, find the equations of the tangents to the ellipse $4x^2 + 9y^2 = 1$ which are perpendicular to the line $y = 2x + 3$. (12 marks)
Hint: Use the discriminant $B^2 - 4AC$ and the condition for tangency. Recall that for perpendicular lines $m_1 \times m_2 = -1$.

11. (a) Investigate the stationary points on the curve $y = x^2e^{-x}$.
Hint: Use condition for stationary points and the fact that e^x is never zero($e^x \neq 0$). Then use the the table or second derivative method to distinguish the stationary points.

(b) Find the area bounded by the curves $y = 15 - 3x^2$ and $y - 3 = 0$. What is the volume of the solid of revolution formed when this area is rotated through 360^0 about the $x-$axis. *(12 marks)*
Hint: Determine upper and lower limits for your definite integral and recall the fact that roation is about the axis.

12. The diagram shows an arrow embedded in a target. The line of the arrow passes through the point $A(2, 3, 5)$ and has direction vector $3\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. The arrow intersects the target at the point B. The plane of the target has equation $x + 2y - 3z = 4$. The units are metres.



(a) Write down the vector equation of the line of the arrow in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$.
Hint: $\mathbf{p}$ and $\mathbf{q}$ is given in the question.

(b) Find the value of λ which corresponds to B. Hence write down the co−ordinates of B.
Hint: Substitute the line in the plane via parametric equations of the line then solve for λ.

(c) The point C is where the line of the arrow meets the ground, which is the plane $z = 0$. Find the co−ordinates of C. *(12 marks)*
Hint: Use method in 12(b).

13. Integrate the following functions

(a) $x^3 e^{-x^2}$. *(05 marks)*
Hint: Use coexistence of a function and its derivative and then apply integration by parts.

(b) $\dfrac{x}{(4-x)^2}$. *(07 marks)*
Hint: Use partial fraction.

14. (a) In how many ways can letters of the word $DETERMINATION$ be arraned if the vowels must not be together and the constonants must not be together. (*06 marks*)

Hint: Use the complement.

(b) Determine the number of odd numbers greater than 700,000 that can be formed using the digits 5, 6, 7, 8, 9 and 0 if no repeatitions are allowed. (*06 marks*)

Hint: Use table method and the condition for odd numbers.

15. (a) Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$ where α is an acute angle. Hence solve $4\sin\theta - 3\cos\theta = 3$. (*05 marks*)

Hint: Use compound angle formula for sin and compare coefficients.

(b) Find the greatest and least value of $\dfrac{1}{4\sin\theta - 3\cos\theta + 6}$ stating the values of θ between 0 and 360^0 at which the occur. (*07 marks*)

Hint: Use maximization and minimization of rational functions.

16. (a) Find positive integers a and b for which $x^4 + 2x^2 + 9 = (x^2 + a)^2 - b^2x^2$ and hence find the quadratic factors of $x^4 + 2x^2 + 9$.

Hint: Use laws of indices and Shreedharacharya's rule.

(b) Solve the simultaneous equations
$x = \dfrac{x+y}{3} = \dfrac{x-y+z}{2}$, $x^2 + y^2 + z^2 + x + 2y + 4z - 6 = 0$. (*12 marks*)

Hint: Apply method of intersection of line and 'plane'.

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