GAYAZA HIGH SCHOOL

S4 MATHEMATICS (MAY)2020

Mensuration is the branch of mathematics which studies the measurement of the **geometric shapes** and the calculation of their parameters like area, perimeter, surface area, volume etc.

Types of Mensuration

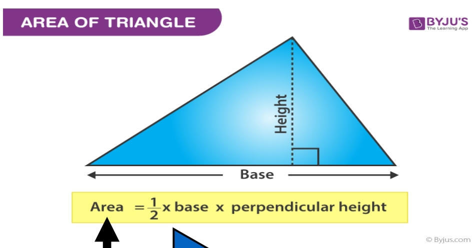
- **Plane mensuration** deals with the sides, perimeters and areas of plane figures of different shapes.
- **Solid mensuration** deals with the surface areas and volumes of solid objects. The shapes exist in either 2 dimensions (2D) or 3 dimensions (3D).

Differences between 2D and 3D shapes

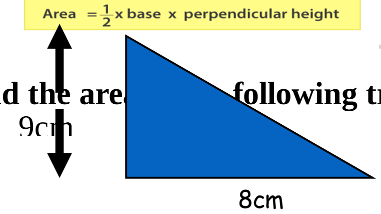
2D shape	3D shape
• This is a shape surrounded by three or more straight lines in a plane. • These shapes are plane figures such as the triangle, square, rectangle, trapezium, parallelograms , rhombus ,kite , circle etc. • These shapes have lengths in two directions . • We can measure and calculate their area and perimeter	• This is a shape surrounded by a number of surfaces or planes. • These shapes are called solids such as the prisms(cube,cuboid, cylinder , triangular , etc),cone,sphere , pyramids etc • These shapes have lengths in three different directions. • We can measure and calculate their volume and total surface area.

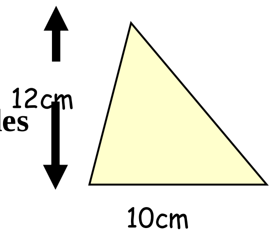
AREA OF TRIANGLE:

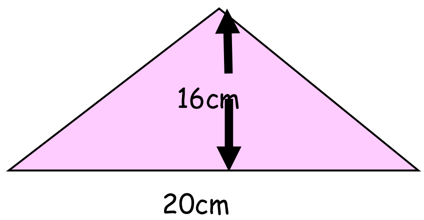
Area of triangle when given the base and the perpendicular height.

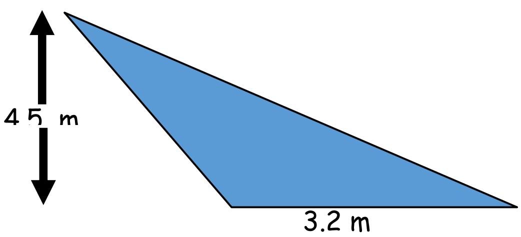


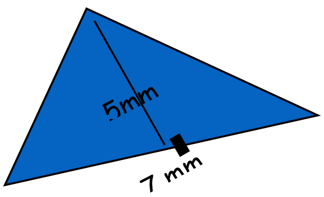
Find the area of the following triangles











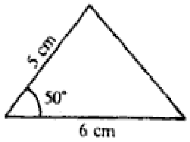
In addition to the formula $Area\ of\ triangle = \frac{1}{2}bh$, there are two other useful formulae

Area of a triangle when two sides and an included angle are given

$$Area\ of\ triangle = \frac{1}{2}bcsinA = \frac{1}{2}acsinB = \frac{1}{2}absinC$$

Example

Find the area of the triangle



$$Area = \frac{1}{2} \times 5 \times 6 \times sin50° = 11.49cm^2$$

Example

Find the area of a triangle ABC such that $AC = 6cm$, $BC = 9cm$ and < BCA = 32°

Solution

$$A = \frac{1}{2}absinC$$
$$= \frac{1}{2} \times 6 \times 9 \times sin32°$$
$$= 14.31cm^2$$

Example

The area of a triangle is $18.1cm^2$, if the two of its sides are 7cm and 9cm, find the included angle .

Solution

$$A = \frac{1}{2}absinC$$
$$18.1 = \frac{1}{2} \times 7 \times 9 \times sin\theta$$
$$sin\theta = 0.5746$$
$$\therefore \theta = sin^{-1}0.5746 = 35.07°$$

Area of a triangle when all the three sides are given

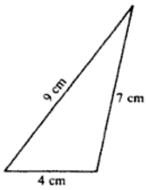
We use Heron's formula

Given that a, b and c are the sides of the triangle ABC, then

$$area\ of\ triangle = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

Example

Find the area of the triangle



Solution $s = \frac{1}{2}(a+b+c) = \frac{1}{2}(9+4+7) = 10cm$

$$Area\ of\ triangle = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{10(10-4)(10-9)(10-7)}$$

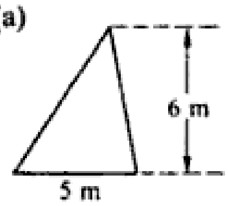
$$= \sqrt{180}$$

$$= 13.41cm^2$$

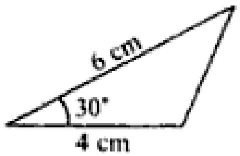
EXERCISE

Find the areas of the following triangles:

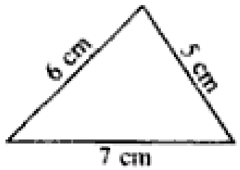
(a)



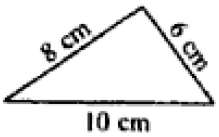
(b)



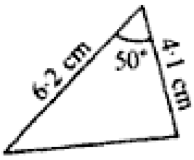
(c)



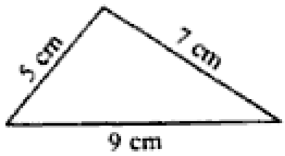
(d)



(e)



(f)



2. In a fitness exercise, students run round the three sides of a triangular field PQR. Given that $PQ = 95m, QR = 120m$ and $PR = 145m$. Find the area of the field.

3. A fishing boat travelled 3.2km from a lighthouse L to a point M. It then travelled from M to a point N and then back to the lighthouse. If $MN = 1.7km,\ LN = 2.8km$, find the area covered by the boat.

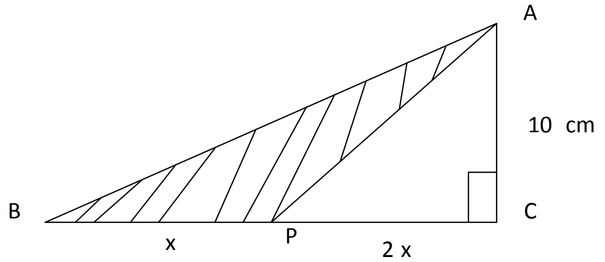
4. A farmer marks off a triangular piece of land for growing vegetables. Given that $AB = 39.5m, BC = 68.6m$ and $< ABC = 43°$, calculate the area of the piece of land.

5. In an isosceles triangle ABC in which $AB = 12cm$, $AC = BC = xcm$ and angle $ACB = 120°$. Find the (a) value of x (b)area of the triangle

Further worked examples

Example 1

In the given triangle ABC, the shaded area is 20cm^2. Given that AC = 10cm, BP = xcm and PC = 2x cm , find the area of the unshaded region.



Solution

Shaded area =area of ABC- area of ACP

$$20 = \left[\frac{1}{2}(3x) \times 10\right] - \left[\frac{1}{2}(2x) \times 10\right]$$

$$20 = 15x - 10x = 5x$$

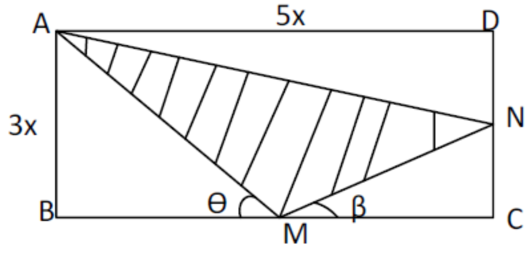
$$5x = 20$$

$$\therefore x = 4cm$$

Area of unshaded region = area of ACP=$10x = 10 \times 4 = 40cm^2$

Example 2

In the figure ABCD is a rectangle in which AD= 5x cm and AB= 3x cm. M and N are the mid points of BC and CD respectively.



(a) Show that the area of the shaded region is $\frac{45x^2}{8}cm^2$

(b) Given that the area of the triangle MNC= $30cm^2$, find the dimensions of the rectangle

(c) Calculate the size of angles;
 (i) θ
 (ii) β
 (iii) AMN

Solution

(a) $Total\ unshaded\ area = area\ of\ \Delta ABM + area\ of\ \Delta MNC + area\ of\ \Delta ADN$

$$= \left[\frac{1}{2}(3x)\left(\frac{5}{2}x\right)\right] + \left[\frac{1}{2}\left(\frac{3}{2}x\right)\left(\frac{5}{2}x\right)\right] + \left[\frac{1}{2}(5x)\left(\frac{3}{2}x\right)\right]$$

$$= \frac{15x^2}{4} + \frac{15x^2}{8} + \frac{15x^2}{4}$$

$$= \frac{75x^2}{8}\ cm^2$$

$$\Rightarrow Area\ of\ shaded\ region = area\ of\ rectangle\ ABCD - total\ unshaded\ area$$

$$= (5x \times 3x) - \left[\frac{75x^2}{8}\right]$$

$$= 15x^2 - \frac{75x^2}{8} = \frac{120x^2 - 75x^2}{8}$$

$$\therefore \ Area\ of\ shaded\ region = \frac{45x^2}{8} cm^2$$

$(b)\ \ Area\ of\ \Delta MNC = \frac{15x^2}{8} = 30\ \Rightarrow x^2 = 16 \quad \therefore x = 4cm$

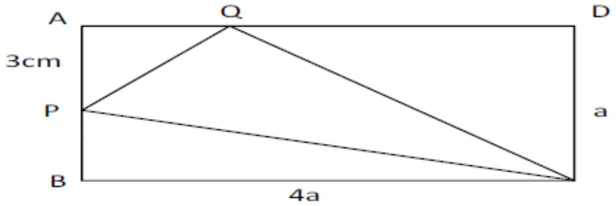
Dimensions of the rectangle are $5 \times 4 = 20cm$ and $3 \times 4 = 12cm$

$(c) tan\theta = \frac{AB}{BM} = \frac{12}{10} = 1.2 \Rightarrow \theta = tan^{-1}1.2 = 50.19°$

EXERCISE

1.

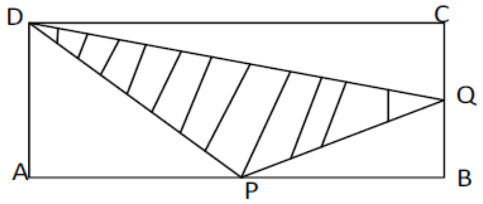
In the diagram below ABCD is a rectangle in which $BC = 4acm$ and $CD = acm$. P and Q are points on AB and AD respectively, such that $AP = AQ = 3cm$



(i) Find the sum of the areas of triangles BCP and CDQ in terms of a
(ii) Given that the area of triangle PQC is $40.5cm^2$, find the value of a
(iii) Express the area of triangle PCQ as a ratio of the area of the rectangle ABCD

2.

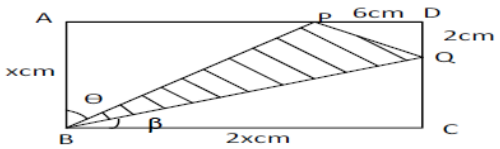
In the figure ABCD is a rectangle in which BC$= 2x\ cm$ and AB$= 3x\ cm$. P and Q are the on AB and BC respectively such that $AP = \frac{3}{4}AB$ and $BQ = \frac{2}{3}BC$



(a) Show that the area of the triangle APD is equal to the area of triangle PQD
(b) Given that the area of the shaded region is $36cm^2$, determine the value of x and state the dimensions of the rectangle
(c) Calculate the size of angle ADP;

3.

The figure below shows a rectangle ABCD in which $AB = xcm$ and $BC = 2xcm$. Points P and Q are on AD and CD respectively such that $PD = 6cm$ and $DQ = 2cm$



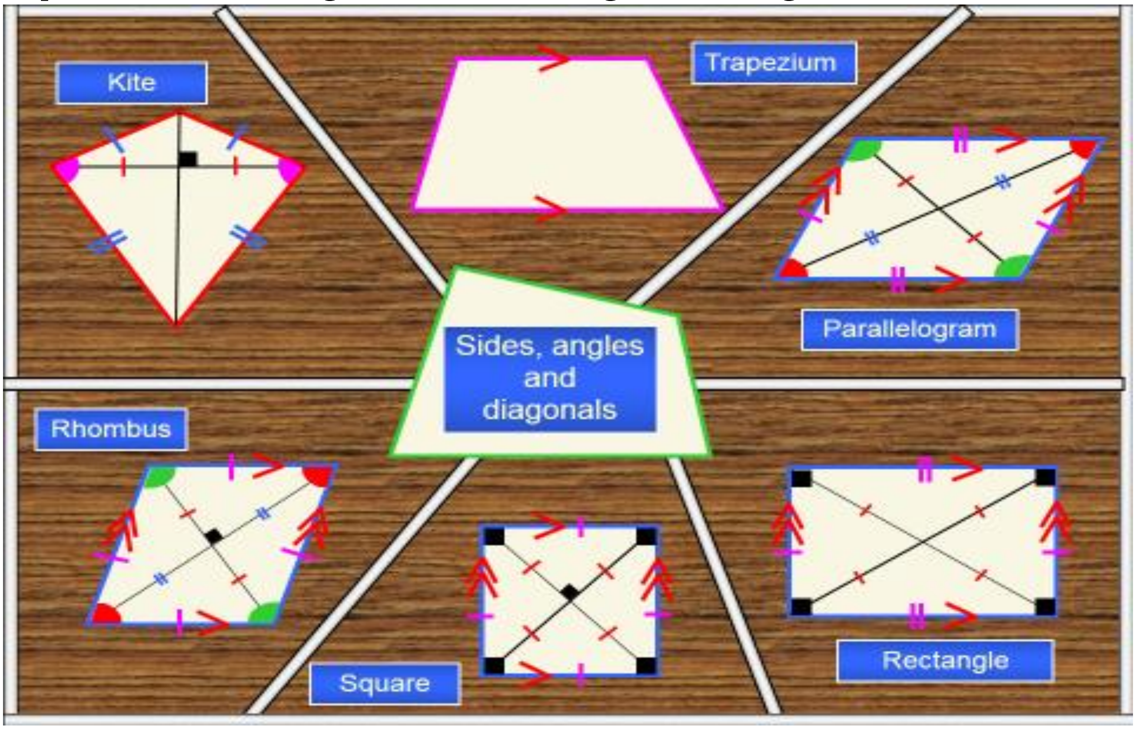
(a) Show that the area of the shaded region is $(5x - 6)cm^2$
(b) Given that the area if triangle ABP is $40cm^2$, find the value of x and hence calculate the area of the shaded region
(c) Find the size of the marked angles

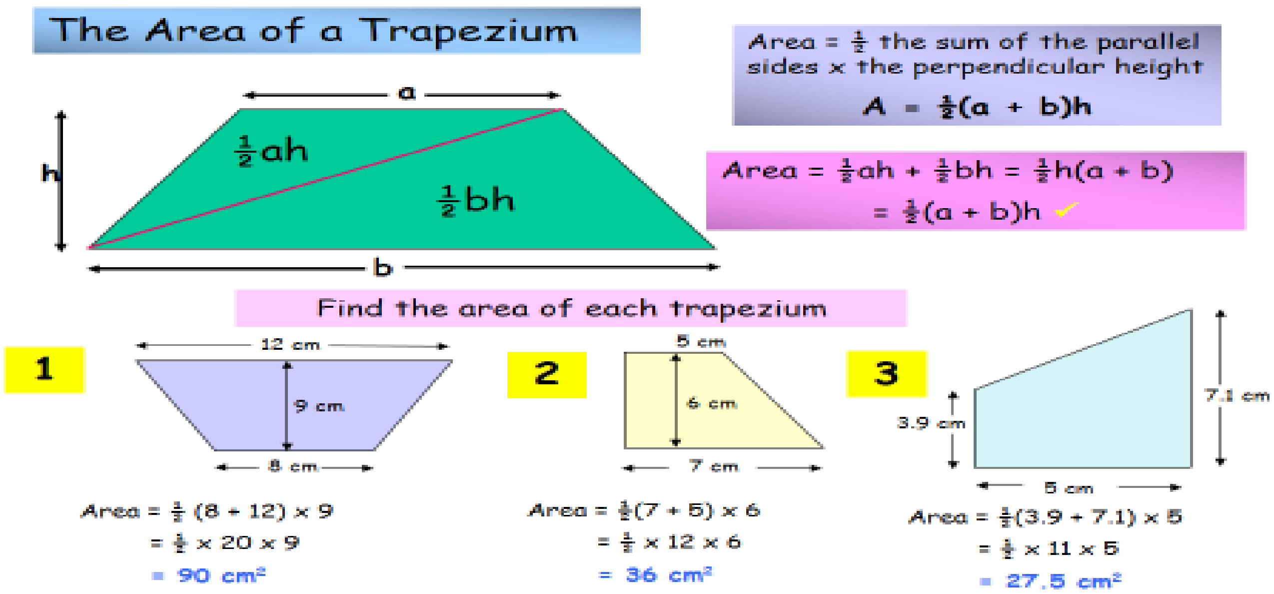
Area of Quadrilaterals

A quadrilateral is a plane figure bounded by four line segments.Examples include square,parallelogram, kite, trapezium,rhombus, rectangle etc .

Activity: PROPERTIES OF QUADRILATERALS

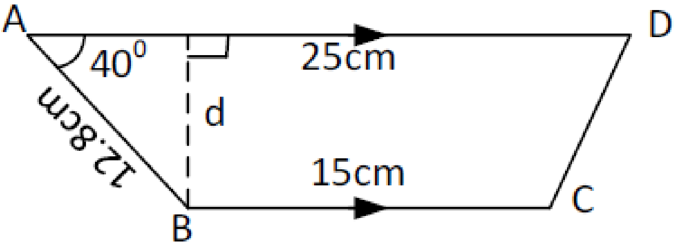
Study the picture below and write down the properties of each of the quadrilaterals basing on their sides, angles and diagonals,





Example

ABCD is a trapezium in which AD is parallel to BC. Given that $AD = 25cm$, $BC = 15cm$, $AB = 12.8cm$ and angle $DAB = 40°$. Calculate the area of the trapezium.

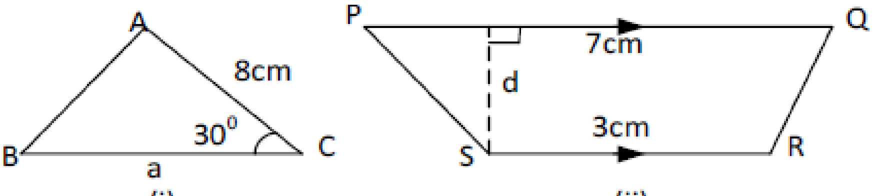


Solution $sin40° = \frac{d}{12.8}$ $\Rightarrow d = 12.8sin40° = 12.8 \times 0.6428 = 8.2278cm$

$$Area\ of\ the\ trapezium = \frac{1}{2} \times 8.2278(15 + 25) = 164.556cm^2$$

Example

Figure(i) shows a triangle ABC in which $AC = 8cm, BC = acm$ and angle $ACB = 30°$. Figure (ii) shows a trapezium PQRS in which $PQ = 7cm, SR = 3cm$, PQ is parallel to SR and the distance between them is $d\ cm$. Given that the triangle and trapezium have the same area , determine the ratio of $d: a$.



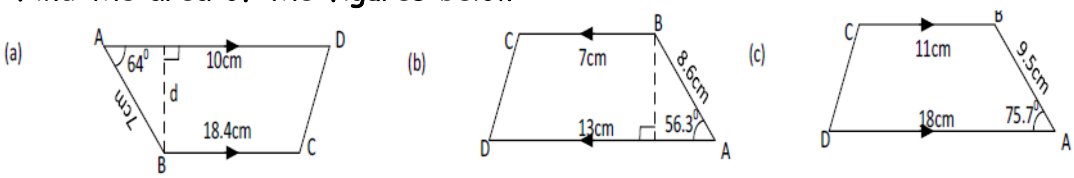
(i) (ii)

Solution

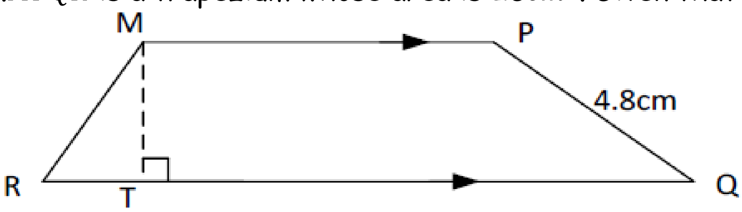
Area of triangle$= \frac{1}{2}8asin30 = 2a$ Area of trapezium$= \frac{1}{2}(7 + 3)d = 5d$ But the triangle and trapezium have the same area $\Rightarrow 5d = 2a \Rightarrow \frac{d}{a} = \frac{2}{5} \therefore d: a = 2: 5$

Exercise

1. **Find the area of the figures below**



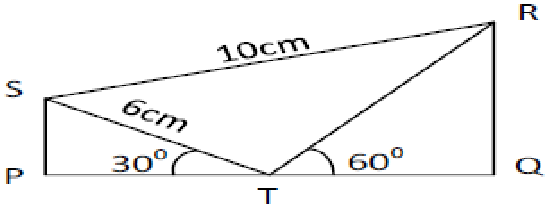
2. $MPQR$ is a trapezium whose area is $25cm^2$. Given that $MP = 6cm, PQ = 4.8cm$ and $RQ = 8.4cm$.



Find (i) PT (ii) angle MPQ

3. The longer side of a trapezium is three times as long as the shorter parallel side. The perpendicular distance between the parallel sides is 15cm. If the area of the trapezium is $180cm^2$.Calculate the length of its longer parallel side.

4. The diagram below shows a trapezium PQRS in which PS is parallel to QR, $SR = 10cm$ and angle $PQR = 90°$.
T is a point on PQ such that $ST = 6cm$, angle $PTS = 30°$and angle $QTR = 60°$.



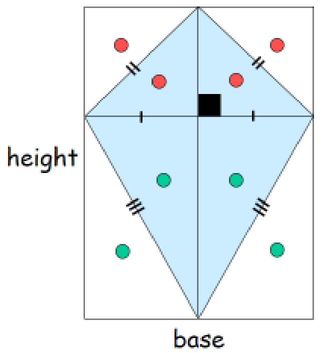
(a) Find the size of angle STR

(b) Calculate the length of (i) TR (ii) QR (iii) PS (iv) PQ

(c) Determine the area of the trapezium PQRS

The Area of a Kite

The area of a kite is ½ the product of its diagonals.



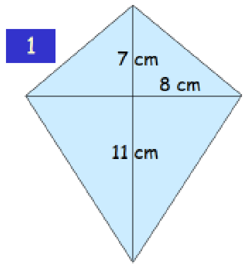
Area of rectangle = 4● + 4●

Area of kite = ½ area of rectangle.

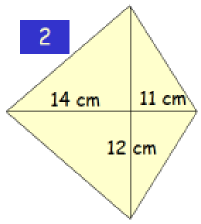
Area of kite = ½ base x height = ½ the product of the diagonals.

The Area of a Kite

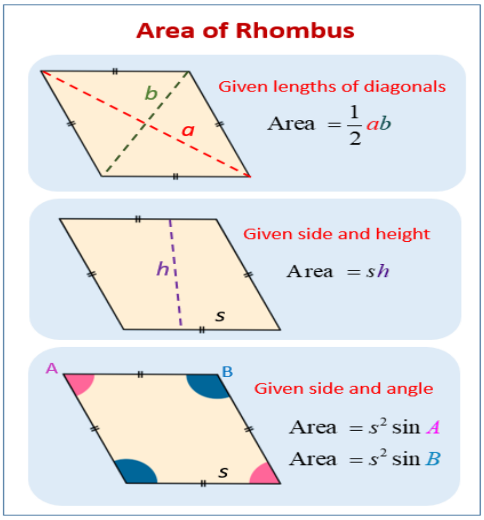
Find the area of the kites below.

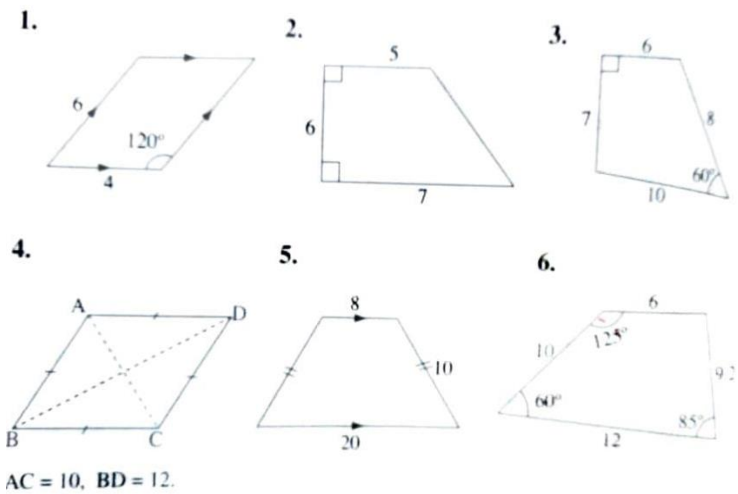


Area = ½ x 16 x 18 = 144 cm²



Area = ½ x 24 x 25 = 300 cm²

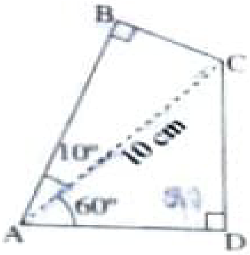




Qns 1-6 Find the area of the figures above

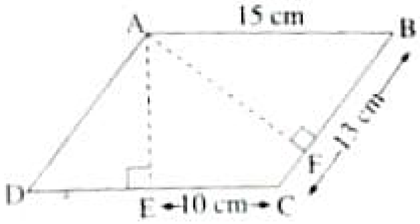
Additional Exercise

7. In the quadrilateral ABCD, $<B =< D = 90°, < BAC = 10°$ and $< CAD = 60°$. Given



that $AC = 10cm$, find the (i) length of AD (ii) length of AB (iii) area of the quadrilateral

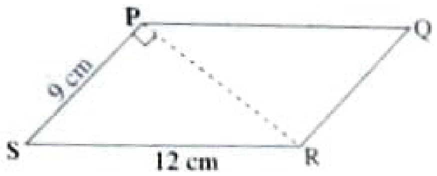
8. (a)Given that $AB = 15cm$, $BC = 13cm, CE = 10cm$ Calculate the area of the



parallaelogram ABCD

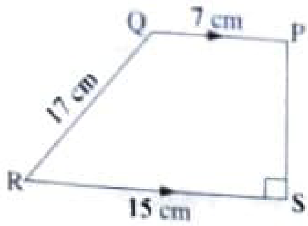
(b) find the length of the perpendicular from A to BC.

9. In the parallelogram PQRS, $< SPR = 90°, PS = 9cm$ and $SR = 12cm$. Find the length



of the diagonal PR. Hence find the area of the parallelogram.

10. PQRS is a trapezium in which $PQ = 7cm, QR = 17cm, RS = 15cm$ and $< PSR = 90°$



and PQ is parallel to SR. Calculate the (i) length of PS (ii) area of the trapezium.