OUR LADY OF AFRICA S.S NAMILYANGO (OLAN)

A LEVEL PURE MATHEMATICS P425/1 SEMINAR QUESTIONS 2019

ALGEBRA

1. (a) The remainder when the expression $x^3 - 2x^2 + ax + b$ is divided by $x - 2$ is five times the remainder when the same expression is divided by $x - 1$, and 12 less than the remainder when the same expression is divided by $x - 3$. Find the values of a and b

   (b) Prove by induction that $8^n - 7n + 6$ is divisible by 7 for all $n \geq 1$.

2. (a) Solve for x : $\dfrac{(1-x)^2}{2-x^2} = \dfrac{(1-a)^2}{2-a^2}$

   (b) Solve the equation: $\log_{10} e.In(x^2+1) - 2\log_{10} e.Inx = \log_{10} 5$

   (c) Given that the first three terms in the expansion in ascending powers of x of $(1+x+x^2)^n$ are the same as the first three terms in the expansion of $\left(\dfrac{1+ax}{1-3ax}\right)^3$, find the value of a and n

   (d) Find the term independent of x in the binomial expansion of $(3x - \frac{2}{x^2})^9$

3. (a) Find x if $\log_5 2$, $\log_5(2^x - 3)$, $\log_5\left(\frac{17}{2} + 2^{x-1}\right)$ form an A.P.

   (b) The first, second, third and n th terms of a series are 4, -3, -16 and $(an^2 + bn + c)$ respectively. Find a, b, c and the sum of n terms of the series.

   (c) The coefficients of the 5th, 6th and 7th terms in the expansion of $(1+x)^n$ are in an A.P. Find n.

4. (a) Show that $z + 2i$ is a factor of $z^4 + 2z^3 + 7z^2 + 8z + 12$, hence solve the equation $z^4 + 2z^3 + 7z^2 + 8z + 12 = 0$.

   (b) Show the locus of arg (z - i) = $\dfrac{3\pi}{4}$ on the Argand diagram and hence or otherwise find the Cartesian equation of the locus.

   (c) If $x, y\ a\ and\ b$ are real numbers and if $x + iy = \frac{a}{b+\cos\theta + i\sin\theta}$, show that $(b^2-1)(x^2+y^2) + a^2 = 2abx$

5. A curve is given by $y = \dfrac{2(x-2)(x+2)}{2x-5}$

   (i) Determine the turning points on the curve and hence find the range of values of y for which the curve is undefined.

   ii) Determine the asymptotes to the curve.
   iii) Sketch the curve.

ANALYSIS

6. (a) A curve is given parametrically by $x = 2\cos\theta + \cos 2\theta, y = 2\sin\theta - \sin 2\theta.$
Show that the gradient at the point parameter θ is $-\tan\frac{1}{2}\theta$ and that the equation of the tangent to the curve at this point is $x\sin\frac{1}{2}\theta + y\cos\frac{1}{2}\theta = \sin\frac{3}{2}\theta.$

(b) If $x^2 + 2xy + 3y^2 = 1$ prove that $(x+3y)^3\dfrac{d^2y}{dx^2} + 2 = 0$

7. (a) Show that the particular solution to the equation $x - y\dfrac{dy}{dx} = y^2\dfrac{dy}{dx} + xy$, for $y(0) = 2$, is $x^2 + (y-2)(y+6) + 4In(y-1) = 0$.

(b) According to Newton's law of cooling, the rate of cooling of a body in air is proportional to the difference between the temperature of the body and that of air. If the air temperature is kept at $25^o C$ and the body cools from $95^o C$ to $60^o C$ in 25 minutes, in what further time will the body cool to $32^o C$?

8. (a) Prove that $\int_1^3 \left(\dfrac{3-x}{x-1}\right)^{1/2} dx = \pi$ (hint: Use the substitution $x = 3\sin^2\theta + \cos^2\theta$

(b) Evaluate: i) $\int_0^1 \dfrac{6x}{x^3+8}dx$ ii) $\int_0^3 x\log_e(1+x)dx$ (iii) $\int_0^{\frac{a}{2}} x^2\sqrt{(a^2-x^2)}dx$

(iv) $\int_0^{\pi/2}\dfrac{dx}{2+\cos x}$ (v) $\int_0^1 \dfrac{x^2}{\sqrt{4-x^2}}dx$ (vi) $\int_0^1 \dfrac{e^x}{(1+e^x)}dx$, use $t = e^x$

(vii) $\int_0^{\pi/2}\dfrac{1}{5\cos x + 4}dx$, use $t = \tan\dfrac{x}{2}$ (viii) $\int_1^2 \dfrac{dx}{\sqrt{12+8x-4x^2}}$

9. (a) Solve the differential equation $\frac{dy}{dx} = \frac{sin^2 x}{y^2}$, if $y = 1$ $when\ x = \pi.$

(b) A machine depreciates at a rate proportional to its current value. Initially the machine is valued at Shs. 2.5 million, 5 years later it was valued at shs. 1.875millions. If θ is the value of the machine after t years, form a differential equation and solve it to find;

(i.) the value of the machine after 15 years

(ii.) the number of years it will take the machine to be valued shs. 0.5 million.

10. (a) Prove that the function $y = \frac{sinxcosx}{1+2sinx+2cosx}$ has turning points in the range $0 \le \theta \le 2\pi$ and then distinguish between them

(b) Show that the tangents at the origin and at the point $(\frac{\pi}{2}, 0)$ meet at a point whose abscissa is $\frac{\pi}{4}$

TRIGONOMETRY

11. (a) Prove that: i) $\tan^{-1}\dfrac{\sqrt{3}}{2} + \tan^{-1}\dfrac{\sqrt{3}}{5} = \dfrac{\pi}{3}$ (ii) $\tan^{-1}\dfrac{1}{2} - \cos ec^{-1}\dfrac{\sqrt{5}}{2} = \cos^{-1}\dfrac{4}{5}$

(b) Solve the following equations,

(i) $tan^{-1}\left(\frac{x-5}{x-1}\right) + tan^{-1}\left(\frac{x-4}{x-3}\right) = \frac{\pi}{4}$ (ii) $sin^{-1}\left(\frac{x}{x-1}\right) + 2tan^{-1}\left(\frac{1}{x+1}\right) = \frac{\pi}{2}$

12. (a) Show that $3cos\theta + 2sin\theta$ may be written in the form $\sqrt{13}\cos(\theta - \alpha)$ where $tan\alpha = \frac{2}{3}$ hence find the maximum and minimum values of the function giving corresponding values of θ.

(b) Prove that if $\tan x = k\tan(A-x)$, then $\sin(2x-A) = \dfrac{k-1}{k+1}\sin A.$. Find all the angles for $0^o \le x \le 360^o$ which satisfy the equation.

13. (a) Simplify $\frac{sin105^0 - \sin(-15^0)}{cos105^0 + \cos(-15^0)}$ giving your answer in the form $R\sqrt{3}$

(b) Given that $x = \tan\theta - \sin\theta$ and , $y = \tan\theta + \sin\theta$. Prove that $(x^2 - y^2)^2 = 16xy$

(c) Prove that $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$, and $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ and hence solve the equation $t^4 + 4t^3 - 6t^2 - 4t + 1 = 0$.

14. (a) Solve $5\sin(x + 60^0) - 3\cos(x + 30^0) = 4$ for $0^0 \le \theta \le 2\pi$

(b) Find all the angles between 0^0 and 180^0 for $\frac{2}{cos^2 2x} - 4 = 3tan2x$

(c) Find the angle B in the triangle ABC where $a = n^2 - 1, b = n^2 - n + 1, and\ c = n^2 - 2n$

VECTORS

15. (a) The point $C(a,\ 4,\ 5)$ divides the line joining $A(1,\ 2,\ 3)$ and $B(6,\ 7,\ 8)$ in the ratio $\lambda : 3$. Find a and λ.
(b) Show that the lines $r = (-2i + 5j - 11k) + \lambda(3i + j + 3k)$, $r = (8i + 9j) + t(4i + 2j + 5k)$ intersect, Hence;
(i) find the position vector of their point of intersection.

(ii) Find also the Cartesian equation of the plane formed by these two lines.

16. (a) Determine the equation of the plane through the points $A(1,\ 1,\ 2)$, $B(2,\ -1,\ 3)$ and $C(-1,\ 2,\ -2)$.
(b) A line through the point $D(-13,\ 1,\ 2)$ and parallel to the vector $12\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ meets the plane in (a) at point E. Find:

i) the coordinates of E.
ii) The angle between the line and the plane.

17. (a) The points A, B, C and D have coordinates $(-7,\ 9)$, $(3,\ 4)$, $(1,\ 2)$ and $(-2,\ -9)$ respectively. Find the vector equation of the line PQ where P divides AB in the ratio $2:3$ and Q divides CD in the ratio $1:-4$.

(b) The planes m and n are given by equations $3x + 2y + z = 4$ and $2x + 3y + z = 5$ respectively.

The plane π containing the point A(2, 2, 1) is perpendicular to each of the planes m and n. Find:

(i) Distance from the point A to the plane m.

(ii) Angle between the planes m and n.

(iii) Cartesian equation of the plane.

(iv) Equation of the line of intersection of the planes m and n.

GEOMETRY

18. (a) Find the equations to the lines through the point (2, 3) which makes angles of 45° with the line $x - 2y = 1$.

(b) A circle with centre P and radius r touches externally both the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x + 8 = 0$. Prove that the $x-$coordinate of P is $\frac{r}{3} + 2$.

19. (a) $ABCD$ is a square; A is the point $(0,\ -2)$ and C is the point $(5,\ 1)$, AC being the diagonal. Find the equations of the lines AB and BC.

(b) The line $y = mx$ and the curve $y = x^2 - 2x$ intersect at the origin O and meet again at a point A. If P is the midpoint of OA, find the locus of P.

20. (a)(i) Find the equation of the tangent to the parabola $y^2 = 4ax$ at point $T(at^2, 2at)$.

(ii) Determine the equations of the tangents to the parabola $y^2 = 6x$ from the point $(2,\ 4)$.

(b)(i) If the tangents at points P and Q on the parabola $y^2 = 4ax$ are perpendicular, find the locus of the mid-point of PQ.

(ii) The tangent to the parabola $y^2 = 4ax$ at point $P(ap^2, 2ap)\ and\ Q(aq^2, 2aq)$ intersect at R. Find the coordinates of R.

(c) A curve is given parametrically by $x = 3\left(\frac{1}{p^2} + \frac{2}{p} + 1\right)\ and\ y = 6\left(\frac{1+p}{p}\right)$ .Show that the curve is a parabola and find its focus.

21. (a)The line $y = x - c$ touches the ellipse $9x^2 + 16y^2 = 144$. Find the value of c and the coordinates of the point of contact.

(b) Prove that the equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point P (a$\sec\theta$,b$\tan\theta$) is a$x\sin\theta$ +by = $(a^2 + b^2)\tan\theta$.

(c) Show that the area of the triangle formed by any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with its asymptote is A=ab square units.

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