SENIOR SIX MATHEMATICS SEMINAR TO BE HELD ON 27TH FEBRUARY 2021 ONLINE SEMINAR THROUGH ZOOM.

PURE MATHEMATICS P425/1

Instructions:

As a school, please select several questions and prepare presentations in each. We shall book the questions of choice through the WhatsApp group and on the seminar day.

VECTORS

1. The lines l_1 and l_2 have equations $l_1: r = 6i + 5j + 4k + \lambda(i + j + k)$ and $l_2: r = 6i + 5j + 4k + \mu(4i + 6j + k)$.
   (i) Find the cartesian of the plane Π containing l_1 and l_2.
   (ii) Find the position vector of the foot of the perpendicular from the point with position vector $i + 10j + 3k$ to the plane Π.
   (iii) The line l_3 has equation $r = i + 10j + 3k + t(2i - 3j + k)$. Find the shortest distance between l_1 and l_3.
2. The lines l_1 and l_2 have vector equations $r = 4i - 2j + \lambda(2i + j - 4k)$ and $r = 4i - 5j + 2k + \mu(i - j - k)$ respectively.
   (i) Find the angle between the lines l_1 and l_2.
   (ii) Show that l_1 and l_2 intersect and find the point of intersection.
   (iii) Find the perpendicular distance from the point P whose position vector is $3i - 5j + 6k$ to the plane containing l_1 and l_2.

ANALYSIS/CALCULUS:

1. The curve C has equation $y = \frac{x(x+1)}{(x-1)^2}$.
   Find
   (a) (i) the asymptotes of C.
   (ii) the point P where the curve C intersects with the asymptotes.
   (iii) the nature of the stationary points of C if they exist.
   (b) Draw the sketch of curve C.
2. (i) Apply a suitable substitution to simplify and evaluate
   $\int_0^1 \frac{\sqrt{x}}{2-\sqrt{x}} dx$

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(ii) Evaluate $\int_0^{\frac{\pi}{2}}(x-1)\cos 2x\,dx$

3. (a) Find

$$\int \frac{8^{\sqrt{x}}}{\sqrt{x}}\,dx$$

(c) The Area enclosed by the curve $y = e^x$, the y-axis and the line $y = 4$ is rotated through 360^o about the x-axis. Prove that the volume, V of the solid generated is $V = e^4 \ln 4 - \frac{15\pi}{2}$.

4. Express $\frac{2x^2-9x\pm6}{x(x^2-x-6)}$ in partial fractions.

Hence, evaluate $\int_4^6 \frac{2x^2-9x\pm6}{x(x^2-x-6)}\,dx$

5. A pond covers an area of $300\ m^2$. A specimen of pondweed grows on the surface of the pond. At time t days after the weed is first discovered, it covers an area of $A\ m^2$. The area of the pond covered by the weed increases at a rate which is proportional to the square root of the area of the pond already covered by the weed. Initially, the area covered by the weed is $0.25\ m^2$ and its rate of growth per day is $1\ m^2$.
   (i) Form a differential equation to show the relationship between A and t.
   (ii) Deduce to the nearest day, the time taken for the pond's surface area to be completely covered by the weed.

GEOMETRY:

1. (a) Three circles of radius $\frac{3}{2}$ have their centres at the points $(3,0), (3,5)$ and $(0,5)$. Prove that the equation of the circle which cuts all three orthogonally is $4x^2 + 4y^2 - 12x - 20y + 9 = 0$.
   (b) Find the equation and radius of a circle passing through the points $A(0,1)$, $B(0,4)$ and $C(2,5)$.

2. (a) Prove that the equations of the tangents to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ with gradient m are the equations $mx - y \pm \sqrt{a^2m^2 + b^2} = 0$.
   (b) Find the locus of the point of intersection of the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ that are perpendicular to each other.

3. Find the equation of the tangent and the normal to the curve $xy = c^2$ at the point $P\left(ct,\ {}^{c}/_{t}\right)$. Given that the normal at P meets the curve again at Q, find the coordinates of

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Q. If the tangent at P meets the y-axis at R, find the equation of the locus of M, the mid-point of PR.

TRIGONOMETRY:

1. Solve the equation: $\cos^{-1} 2x - \cos^{-1} x = \frac{\pi}{3}$.

b) Find the maximum and minimum values of the function $\frac{1}{3 + \sin x - 2\cos x}$ stating clearly the values of x.

2. Given that $p = 2\cos\theta + 3\cos 2\theta$ and $q = 2\sin\theta + 3\sin 2\theta$:

i) find the greatest and least values of $p^2 + q^2$.

ii) given that $p^2 + q^2 = 19$, find θ for $0^o \leq \theta \leq 90^o$.

iii) Show that $pq = -\frac{5\sqrt{3}}{4}$.

3. (a) Show that in any triangle ABC, if $2s = a + b + c$,

$1 - \tan\frac{1}{2}A \ \tan\frac{1}{2}B = \frac{c}{s}$.

(b) Prove that $4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$.

ALGEBRA

1. (a) The function $\boldsymbol{f = x^3 + px^2 - 5x + q}$ has a factor $(x - 2)$ and a value of 5 when $x = -3$. Find the values of p and q.

(b) The roots of the equation $\boldsymbol{x^2 + x + 1 = 0}$ are α and β. Form the quadratic equation whose roots are $\boldsymbol{\alpha\beta}$ and $\boldsymbol{\alpha}$.

(c) Simplify: $\frac{\sqrt{3}-2}{2\sqrt{3}+3}$ in the form $\boldsymbol{p + q\sqrt{3}}$ where *p*, *q* are rational numbers.

2. (a) Solve the equation $\log_2 x - \log_8 x = 2$.

b) Solve $\log_x 5 + 4\log_5 x = \log_4 256$.

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c)Solve for x for which $2(3^{2x}) - 5(3^{x}) + 2 = 0$.

(d) Find the solution of the inequlity $\frac{x+1}{x-1} < \frac{x+3}{x+2}$.

3. Simplify:

(a). $\frac{x^2(x^2+1)^{-\frac{1}{2}} - (x^2+1)^{\frac{1}{2}}}{x^2}$.   (b). $\frac{(8)^{\frac{1}{6}} \ X \ (4)^{\frac{1}{3}}}{(32)^{\frac{1}{6}} \ X \ (16)^{\frac{1}{12}}}$ (c). $\left(\frac{16}{81}\right)^{-\frac{1}{4}}$

CURVE SKETCHING

1. Sketch the curve $y = \frac{4(x-3)}{x(x+2)}$ by investigating clearly the horizontal region of inexistence
2. Sketch the curve $y = \frac{4x^2}{(x^2+1)(x-2)}$
3. By clearly stating the turning point and all the asymptotes, ketch the curve $y = \frac{x^3}{x^2-4}$
4. Sketch the curve $y = \frac{3x+3}{x(3-x)}$ clearly find the turning points, asymptotes and the horizontal region where the curve does not exist

Important information:

Format for the presentations:

(a) All schools will receive the same questions and will be required to select questions and book with the organisers their preferred questions for discussion through the WhatsApp platforms.

(b) Each presenter selected to present at the seminar will prepare their presentation and type it out if possible or write it down in a good handwriting with a black pen. These presenters should be supported by their subject teachers where possible. These final presentations will be sent to the organisers to be uploaded on the HeLP site before the seminar day- www.help.sc.ug .

(c) On the seminar day within the schools:

i. Each school will designate a presentation room that will accommodate the team making the presentation and answering questions from the audience. This room will have all the camera work with a fine and long blackboard or white board and a support subject teacher and ICT back stopper.

ii. The school should also arrange for 2-3 other rooms where the rest of the students will sit while observing the health SOPs. Each room should have a projector and laptop connected to the internet with a solid sound system. The school could use a big hall and arrange 3 screens all connected to laptops.

iii. All the teachers of mathematics in the school should be encouraged to attend the seminar to support the students in following keenly the presentations.

iv. On the seminar day, the expert presenter will make a presentation(15min) and answer the questions raised from the audience(10min)

v. The teachers will also make any additional comments(5min)

vi. A time will be provided between presentations to receive comments from UCC, RENU and other partners.

The Holistic eLearning Platform (HeLP) is inviting you to a scheduled Zoom meeting.

Topic: A-LEVEL MATHEMATICS VIRTUAL SEMINAR

Time: Feb 27, 2021 08:30 AM Nairobi

Join Zoom Meeting

https://us02web.zoom.us/j/81410189011?pwd=Rit6YzJCNzA3RzdMOFMrNnJyTUdyZz09

Meeting ID: 814 1018 9011

Passcode: 940152

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